Simulations of Acoustic Tomography Array Performance With Untracked or Drifting Sources and Receivers

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Ocean tomography as originally proposed required all sources and receivers to be tautly moored and acoustically tracked to separate travel time perturbations due to mooring motion from those due to ocean features. It is possible to process the tomographic travel times to estimate both ocean sound speed perturbations and mooring offsets, effecting a separation without external tracking. A side effect of this processing is a check on the ray identification, since the varying instrument positions can be used as a synthetic array for estimating ray angle. Simulations and examples with actual data were used to contrast mapping performance with and without mooring tracking for a variety of ray data sets. In general, the ocean maps degrade when the tracking data are withheld. However, when many high-precision ray travel time measurements are available, the degradation is small; in these cases it would be possible to deploy free-drifting instruments as part of a monitoring experiment.

1. INTRODUCTION

Ocean tomography as realized in a 1981 experiment [Ocean Acoustic Tomography Group, 1982] inferred ocean structure from travel times for acoustic rays between autonomous sources and receivers moored at mid-depth in a 300 x 300 km array. Each instrument had an independent clock and could move in any direction as the mooring leaned in response to currents. Both mooring position shift and clock error ('instrument offsets') can produce measured travel time changes which swamp those due to sound speed variation, so the x, y, z offsets of a mooring from its assumed position and shifts of the instrument's clock from the true time must be either measured directly and removed or filtered out in processing. The 1981 tomography experiment included transponders and frequency standards to measure these offsets and remove them before the acoustic data were processed. The correction data set included gaps and errors, and the mooring anchor positions were not known exactly, which implies a large, constant travel time bias.

It was thus necessary to devise a method for reducing the effects of these "instrument offsets" on the tomographic sound speed estimates so that maps could be made even when the correction systems were not operating. The problem is similar to that of determining earthquake location and seismometer site corrections at the same time that earth structure is estimated. It is possible to use the tomographic travel time data to estimate simultaneously both ocean maps and mooring positions. This paper will present a technique for making these estimates and will use examples to explore trade-offs between independent corrections and acoustic data, to see, for example, what price in map accuracy is paid for the loss of instrument offset corrections. Under certain conditions, mooring tracking is unnecessary, and tomographic equipment could be deployed on drifting floats like the SOFAR floats now in use.

2. TOMOGRAPHY

To fix ideas (and allow the use of real data), I will consider an array of instruments such as was deployed in 1981. The overall configuration and local bottom topography for the experiment is shown in Figure 1. Four sources (S1-S4) on the western side and four receivers, (R1-R4) on the east approximately define a 300 x 300 km box. A fifth receiver, R5, was placed near the northern boundary of the box. The instruments were at depths of about 1400---2000 m on taut moorings with subsurface floats (to minimize motion).

The tomographic data consist of arrival times for sound pulses traveling along distinct paths. A pulse traveling from a source to a receiver through an ocean with sound speed $c(x,y,z,t)$ (ignoring internal waves) will follow one of a number of possible ray paths, $F_k$, determined by that sound speed field. The measured source to receiver travel time $D_k$ (for path $F_k$) is given by

$$D_k(t) = \int_{F_k} \frac{ds}{c(x,y,z,t)} + E_k$$

$s$ is arc length along the ray path, and $E_k$ is a general error term, into which are lumped travel time contributions from clock drift $A_k(t)$, mooring motion $B_k(t)$, and internal waves and uncertainty in peak location $e_k(t)$.

When the ocean sound speed field changes, the travel time will change, and if these changes are small, the expression for travel time (1) can be linearized around a basic state, $c_0(x,y,z)$. Let

$$c(x,y,z,t) \equiv c_0(x,y,z) + c'(x,y,z,t)$$

where $c'$ is small compared to $c_0$. To first order, the expression for travel time perturbation becomes

$$D'_k = D_k - D^0_k = -\int_{F'_{k}} \frac{c'(x,y,z,t)}{c_0^2(x,y,z)} ds + E_k$$

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Paper number SC0417
0148-0227/85/00SC-0417S05.00

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where $\Gamma^C$ is the ray path for the basic state $c_0$, $D^C$ is the travel time for the ray in the basic state, and $D_k$ is the perturbation in observed travel time. Equation (3) is linear, and standard inverse techniques can therefore be used to infer $c'$ given a data set $\{D_k\}$, provided $E_k$ is known or negligible. $E_k$ is not a random error, and by considering the physics of each component the effect on the sound speed estimator can be greatly reduced.

The source clock shift will be the same for all rays which leave that source, and the receiver clock shift will likewise be constant for all rays which hit the same receiver. Clock errors can thus be parameterized in terms of only one number (time dependent) for each mooring, and the effect on a given ray will depend only on which source-receiver pair it belongs to. For a ray $k$ from source $i$ to receiver $j$, the contribution $A_k$ to the measured travel time from clock errors $T_c$ and $T_r$ will be $T_k - T_c$:

$$A_k = T_k - T_i$$

(4)

The clock shifts $T_i$ are independent between instruments, so the cross covariances, $\langle T_i T_j \rangle (i \neq j)$, should be zero, and no further parameterization is necessary. Instead of a white noise variance added to all data, the clock noise can be described by $NS + NR$ parameters (where $NS$ is the number of sources and $NR$ is the number of receivers), greatly reducing the effect of unknown clock error.

The mooring motion noise is also correlated, as can be seen by examining its physical basis. Let $X_i$ be a vector holding the $(X, Y, Z)$ location of instrument $i$. In a perturbation framework, the travel time anomaly $B_k$ (due to mooring motion or anchor position offset) for ray $k$ from source $i$ to receiver $j$ can be written as a linear function of the shifts of the instruments, $X_i$, $X_j$, from the reference positions $X_0, X_0$ assumed a priori.

$$B_k = \frac{\partial D_k}{\partial X_i}X_i' + \frac{\partial D_k}{\partial Y_i}Y_i' + \frac{\partial D_k}{\partial Z_i}Z_i'$$

(5)

The partial derivatives in (5) could be estimated by ray tracing for different coordinates, but a simple perturbation approach allows analytical calculation of these quantities. First, decompose the horizontal terms into two parts: the dependence of travel time on horizontal range $R_k$, and the dependence of horizontal range on the individual $x$ or $y$ coordinate:

$$\frac{\partial D_k}{\partial X_i} = \frac{\partial D_k}{\partial R_k} \cdot \frac{\partial R_k}{\partial X_i}$$

(6a)

$$\frac{\partial D_k}{\partial Y_i} = \frac{\partial D_k}{\partial R_k} \cdot \frac{\partial R_k}{\partial Y_i}$$

(6b)

$$\frac{\partial D_k}{\partial X_j} = \frac{\partial D_k}{\partial R_k} \cdot \frac{\partial R_k}{\partial X_j}$$

(6c)

$$\frac{\partial D_k}{\partial Y_j} = \frac{\partial D_k}{\partial R_k} \cdot \frac{\partial R_k}{\partial Y_j}$$

(6d)

(6)

The partial derivatives of range with respect to horizontal position can be calculated from the simple geometry (see Figure 2). Let $\phi$ be the azimuth (east = 0) from source to receiver, then

$$\frac{\partial R_k}{\partial X_i} = \frac{(X_i - X_j)}{R_k} = -\cos \phi$$

(7a)

$$\frac{\partial R_k}{\partial Y_i} = \frac{(Y_i - Y_j)}{R_k} = -\sin \phi$$

(7b)

$$\frac{\partial R_k}{\partial X_j} = \cos \phi$$

(7c)

$$\frac{\partial R_k}{\partial Y_j} = \sin \phi$$

(7d)

while the dependence of arrival time on range and vertical position can be approximated by assuming that the ray has a finite width, with the phase fronts normal to the ray path, so that the extra travel time resulting from the perturbed instrument position will be the time it takes for the phase front to reach it. If ray path $k$ is assumed to be locally straight near instrument $i$, at an angle $\theta_k$, to the
horizontal, and the local sound speed is \( c_i \) (see Figure 3), then

\[
\frac{\partial D_k}{\partial Z_i} = -\frac{\sin \theta_{ik}}{c_i} \quad (8a)
\]

\[
\frac{\partial D_k}{\partial Z_j} = \frac{\sin \theta_{jk}}{c_j} \quad (8b)
\]

\[
\frac{\partial D_k}{\partial R_k} = \frac{\cos \theta_{ik}}{c_i} \approx \frac{\cos \theta_{jk}}{c_j} = p_k \quad (9)
\]

The dependence of travel time on depth must be calculated at both source \( i \) and receiver \( j \) locations, but the partial of travel time with respect to horizontal range is \( p_k \), the "ray parameter," which is nearly constant along ray \( k \) if the horizontal gradient of sound speed is small. This means that the approximation that travel time is a simple function of horizontal separation is correct but that \( p_k \), not \( c_i \), is the constant of proportionality.

Note that these expressions assume the rays to be identified, so that the angles at both source and receiver are known. Conversely, as the mooring moves, the travel time change for each peak in the arrival pattern will depend on the angle with which it arrives. If the movement of the peak can be followed, then motion of a single instrument over short periods produces a synthetic "array" of positions and arrival times which can be used to estimate local ray angle. These angle estimates are, in turn, useful in ray identification. Vertical motion is most effective at estimating ray angle because of the \( \sin \theta \) dependence, but horizontal motions can contribute, provided that the noise level is small enough. Because the ray angles \( \theta \) are typically <0.20 rad, the vertical motion produces travel time changes about one order of magnitude smaller than those for the same size horizontal motion.

Parameterization reduces the mooring motion errors to three unknowns per mooring. These are presumed to be independent, but if the mooring moved as a rigid rod, there would be only two unknown variables per mooring, lean angle and lean direction. The moorings in general use are flexible enough to require the expression of mooring motion travel time in the form (5). Even without a rigid mooring, there is still an expected correlation between lean and depth change, although its form may not be simple. If this correlation is well known in advance, it may be used to enforce a connection between horizontal excursion and depth, but to date this information has not been available, so the worst case (least knowledge) assumption of no correlation has been used. In any case, three degrees of freedom are necessary to treat non-moored applications of tomography. For example, it allows one to consider outfitting floats with sophisticated transmitters and receivers, adding tomographic information to the simple position calculations now used.

### 3. Including Instrument Offsets in the Estimation Procedure

Once the contributions of sound speed perturbations, mooring motion, and clock error to the travel time have been calculated for each ray, a data-data covariance matrix can be constructed in order to use the stochastic inverse [Aki and Richards, 1980]. The stochastic inverse is a weighted least squares inverse which is equivalent to (but more convenient than) a weighted, tapered singular value decomposition inverse. Objective mapping [Bretherton et al., 1976] is an example of a stochastic inverse.

Let \( G \) be the matrix converting mooring motion and clock error to travel time for each ray, and define \( P^T = [X_1, X_2, \ldots] \) to be the vector of \( x, y, z \) and time

<table>
<thead>
<tr>
<th>Table 1. Assumed Daily Average Mooring Motion Standard Deviation</th>
</tr>
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<tbody>
<tr>
<td>Component of Displacement</td>
</tr>
<tr>
<td>Mooring</td>
</tr>
<tr>
<td>S1</td>
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<tr>
<td>S2</td>
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<td>S3</td>
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<td>R3</td>
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<tr>
<td>R4</td>
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<tr>
<td>R5</td>
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</table>

Units are in meters.

<table>
<thead>
<tr>
<th>Table 2. Assumed Hourly Mooring Displacement Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component of Displacement</td>
</tr>
<tr>
<td>Mooring</td>
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<tr>
<td>S1</td>
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<td>S2</td>
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<td>R3</td>
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<td>R4</td>
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<tr>
<td>R5</td>
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</tbody>
</table>

Units are in meters. This is the expected rms deviation from the mean displacement if the tracking is missing or ignored.
offsets for all the moorings, so that if $D'$ is the vector of travel time anomalies and the ocean is unperturbed ($c' = 0$), then

$$D' = GP + \epsilon$$ \hspace{1cm} (10)

By assumption, each element of $P$ is independent of the others, so the covariance matrix $\langle PP^T \rangle = C_P$ will be diagonal, with each diagonal element reflecting the expected variance of that component on the day under consideration. These expected errors are estimated on the basis of the quality of the corrections available on that day and change from hour to hour. This covariance matrix for the mooring shifts can be used as the column weighting in a singular value inversion. If the stochastic inverse is used, then $C_P$ is needed to construct the data-data covariance matrix $Q_P$ for the mooring shifts:

$$Q_P = G \cdot C_P \cdot G^T$$ \hspace{1cm} (11)

The total covariance matrix for the travel time has three components: variation due to the ocean sound speed changes $Q$, see Cornuelle et al., 1985], the mooring shifts $Q_P$, and the remaining random error which is uncorrelated between rays $C_e$ (diagonal);

$$\langle DD'^T \rangle = Q = Q_o + Q_P + C_e$$ \hspace{1cm} (12)

Since the mooring shifts are now included in the inversion in parameterized form, they can be estimated by constructing the complete stochastic inverse operator:

$$\hat{P} = C_P \cdot G^T \cdot (Q^{-1}) \cdot D'$$ \hspace{1cm} (13)

$$\hat{c}'(x,t) = \langle c'(x,t)D'^T \rangle \cdot (Q^{-1}) \cdot D'$$ \hspace{1cm} (14)

The associated expected errors are

$$\langle (\hat{P} - P)(\hat{P} - P)^T \rangle = C_P \cdot G^T \cdot (Q^{-1}) \cdot G \cdot C_P$$ \hspace{1cm} (15)

$$\langle (c' - c')^2 \rangle = \langle c'^2 \rangle - \langle c'D'^T \rangle \cdot (Q^{-1}) \cdot \langle D'c' \rangle$$ \hspace{1cm} (16)

Some of the data used in the inverse may not be travel times, but in any case, each row of $G$ will express the dependence of that datum on the mooring shifts. For example, a pressure measurement on one of the moorings would measure the vertical motion of that mooring. In fact, the records obtained from the mooring tracking transponders could be used directly as data in the inverse, short circuiting any need for separate calculations in advance. In the limit of maximum rigor, the motion of the water as observed by the acoustics and the current meters would have to be consistent with the mooring motions. These perhaps complex interconnections could be exploited to increase resolution, since the indeterminacy would be reduced by each addition of physical relations, but at some point the resolution gain would not be worth the extra effort required to add the extra physics to the inverse.

4. PROCEDURE

Given the array configuration used in 1981 (Figure 1), three quantities determine the performance of the array: the number of rays available as data, the random error in the travel time measurements for each ray, and the presence or absence of corrections for instrument offsets. In the 1981 data, 96 rays could be usefully matched with numerical ray paths, but many had relatively large travel time measurement errors due to interaction with preceding and following ray arrivals and unresolved but interfering multipath arrivals. The comparison of the tomographic results with other measurements used 58 rays with assumed errors of 5 ms rms for daily averaged travel times, including internal wave noise [Cornuelle et al., 1985].

The potential performance of the 1981 array has been examined using inverse operators for 12 distinct hypothetical data sets, allowing two data set sizes, 60 and 96 rays; three rms travel time errors, 5, 1, and 0.2 ms; and either complete instrument offset corrections or none at all. Two additional inverse operators were constructed for the 1981 data set, using observed noise levels (5 ms), 58 rays, and including or ignoring the measured 1981 instrument offset corrections. Each inverse was performed as outlined schematically by (13) and (14), using a data-data covariance made of components as in (12). $C_e$ was taken to be diagonal, with the individual travel time errors set at 5, 1, or 0.2 ms for the synthetic cases and at levels determined.
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Fig. 5. Synthetic ocean "test pattern" plotted as sound speed anomaly at 700 m depth referenced to the MODE I average sound speed profile. The contour interval is 0.5 m s\(^{-1}\). The left-hand side shows the sound speed anomaly field from two 300-km planetary waves with vertical structures given by the first baroclinic mode (see Figure 4). The right-hand side shows the sound speed anomaly field for two 150 km first baroclinic mode waves, with a superposed spatially uniform first baroclinic mode sound speed field with 2 m s\(^{-1}\) amplitude at 700 m depth. In each case the spatial mean value (0 and \(-2\) m s\(^{-1}\), respectively) has been indicated by a dashed contour.

by the data set (about 5 ms after averaging) in the examples using actual data. The mooring motion covariance \(C_p\) was also assumed to be diagonal. For synthetic cases where the mooring corrections were present, they were assumed to be perfect, meaning \(C_p=0\). For the simulations without mooring data, the rms values were assumed to be those listed in Table 1, roughly based on observations during the 1981 experiment. The 1981 data examples (Yearday 82) used this same \(C_p\) to account for the unknown anchor positions and the arbitrary mooring tracking zero reference. Because the same \(C_p\) was used, the simulations with 60 rays and 5 ms error can be compared to the results for the 1981 data. When mooring tracking was unused or unavailable in the 1981 data, the variances in Table 2 were added to \(C_p\).

The ocean sound speed field was modeled with three uncorrelated vertical modes (Figure 4) corresponding to the first, second, and third baroclinic quasi-geostrophic modes, with energies of 2500, 250, and 100 (m s\(^{-1}\))^2, respectively. Each is assumed to have a Gaussian horizontal covariance with a 100-km decay scale. This was used for both simulated and actual data and is an approximation to the covariance and energies observed in the MODE I experiment [MODE Group, 1978]. The sound speed field was also assumed to have nonzero spatial mean components in each of the modes, with energies of 2500, 1000, and 500 m\(^2\) s\(^{-1}\), respectively. The explicit mean component is necessary to allow for the uncertainty in the assumed reference profile. These expected energies can be converted to sound speed variances by squaring the mode amplitudes at the depth of interest (Figure 4), multiplying by the listed energies, and summing.

Although each inverse calculation includes an estimate of the expected error, (see equations (15) and (16)), these are weighted by the assumed ocean spectrum, and it is useful to also test the inverse reproduction of synthetic data sets generated from assumed ocean sound speed fields. Two fields were generated using the vertical sound speed disturbance profile corresponding to the first baroclinic mode for the MODE I region (Figure 4). The mode was modulated in the horizontal by two planetary wave trains with NW and SW phase propagation, and with wavelengths of 300 and 150 km (Figure 5). The 150-km waves were superimposed on a spatial mean field of infinite extent, with vertical structure also given by the first baroclinic sound speed mode. Sample responses to three scales are thus available, since the mean was estimated in a separate step before being added to the mesoscale estimate. The maps generated using the synthetic data can be compared directly with the assumed fields for particular examples of the statistical performance given by the error maps. No mooring offsets were used in either test data set. A third test data set, travel times from pure instrument offsets, was also used, but the ocean maps made from this data set showed negligible (\(-0.1\) m s\(^{-1}\)) perturbations well within the error bars and are not shown.

5. RESULTS

First, consider the sound speed maps made by applying the 12 hypothetical inverse operators to the ray travel
the other hand, the spatial mean component in the small-scale pattern is about $-2 \text{ m s}^{-1}$ at 700 m and is reproduced in even the uppermost row of Figure 7 (dashed contour).

The ocean contains energy in a range of scales, and the error bars (15) calculate the expected error variance at every point in the volume for the ocean spectrum corresponding to the 100-km Gaussian covariance. These error variances have units of square meters per second but have been converted here to percentages by dividing by the expected variance of the true field, so that 100% error means that the estimate is worthless, while 0% error indicates a perfect estimate. Figure 8 gives the error maps (at 700 m) that correspond to the 12 hypothetical cases, assuming that the ocean has a spectrum like that observed in MODE-I. The presence of receiver R5 results in the lower error values near the top center of each map. The average error for each map is given in Table 3, and the error in the spatial mean has been separated out and is listed in Table 4. The errors for estimates of the spatial mean field are smaller than errors for the mesoscale field and were kept distinct to enhance the differences between cases. Either the error maps in Figure 8 or the numbers listed in Tables 3 and 4 provide a quantitative comparison between cases for the Gaussian ocean. The errors steadily decrease along each row, but the decrease is far slower for the tracked data than for the raw data. The trade-off between tracking and data set quality is apparent in the times computed for the two 300-km waves (Figure 6). Reading across the top row from left to right shows the maps produced for the 60-ray data set without any corrections for instrument offsets. The approximation to the true map (left-hand side of Figure 5) improves as the error in the data is reduced (moving to the right in Figure 6), but it never achieves the accuracy of the tracked data (second row in Figure 6). For the 96-ray data set (third row of Figure 6), good visual accuracy is attained for 1-ms error in the absence of corrections. For this large-scale pattern, the 96-ray, 1-ms error, uncorrected inverse performs about as well as the tracked inverse with 60 rays. In all cases (second and fourth rows) the tracked data adequately reproduce the field.

The tomographic system has a much harder time with the second test pattern (Figure 7). The maps made from the 60-ray data set without tracking are very poor, smearing together adjacent highs and lows, and the pattern does not really begin to emerge until 96 rays with small errors are used. The tracked estimates do not improve strikingly in either row and never completely reproduce the test pattern with the accuracy seen in the large-scale cases. On the other hand, the spatial mean component in the small-scale pattern is about $-2 \text{ m s}^{-1}$ at 700 m and is reproduced in even the uppermost row of Figure 7 (dashed contour).
error estimates; for example, the tracked data with 60 rays and 5 ms travel time error perform about as well as the raw data with 96 rays and only 0.2 ms travel time error. These error maps could be used for array design in the same way that objective maps [Bretherton et al., 1976] are used for conventional mooring layout. For the particular case of array and ocean spectrum covered here, even with 96 rays and 0.2 ms error, the cost in resolution of omitting mooring tracking still amounts to about 10% of the expected mesoscale variance (Table 1), or about 8% of the total energy.

The 1981 data provide a check on the validity of the theory used in the simulations (Figure 9), and it is even possible to examine the tracking trade-off in a primitive way. The error level is fixed, and perfect corrections are unavailable, so the comparison must be between partial corrections and none at all. Finally, the true field is only imperfectly known by limited independent nontomographic measurements, so the statistical error map may be the best performance indicator. The example shown in Figure 9 shows little improvement when the tracking data are used because of the dominance of the mooring anchor position uncertainty.

Inverse estimates of the mooring offsets, $\hat{P}$, and the associated error bars were generated for the raw data runs as part of the mapping procedure for the ocean field, and response to a "test pattern" of mooring displacement was also studied for all untracked cases. The fundamental uncertainty in the mooring parameter estimation problem is the leakage of ocean energy into mooring energy and vice versa. If, for example, an ocean perturbation is present but the moorings are unaffected, the inverse will still estimate new values for the mooring parameters as a result of misinterpretation of the travel time signal (analogous to aliasing in time series analysis). These leakages are the result of indeterminacy in the inverse and are included in the estimated error bars (Table 5). The effects of the mooring movements on the ocean maps are minimal, even for relatively poor quality data sets, because the inverse was designed to be conservative and even reject ocean structures that resembled mooring effects. Consequently, the inverse generally overestimates small mooring shifts. As the data set quality rises, the ability to distinguish ocean from mooring offset improves, and both estimates improve as the leakage drops.

The horizontal mooring position error bars remain large even when data set quality is good (Table 5). This is because the travel times only measure relative displacement within the array, while the error bars include the uncertainties in the absolute locations. If the entire array is uniformly translated in $x$ or $y$, then the instrument spacings are not affected, and no travel time signal is produced (provided the basic ocean state is horizontally invariant). Solid body rotation of the array also does not change the ranges between instruments and so will not affect travel times either (if the basic state is horizontally isotropic).

If there are NS sources and NR receivers, then there are 18 unknowns for horizontal positions. If only travel times are used to estimate these parameters, three components (translation and rotation) remain unknown, and the error bars include the possible errors in absolute position. If the initial uncertainty projected equally in all directions in mooring parameter space, then the three remaining unknowns should have $1/6$ of the initial variance between them. For a single instrument, given the variances of Table 3, the radius of uncertainty would be about 200 m. The minimum radius in Table 5 is 180 m, consistent, within the rough calculations, with nearly pure absolute

### Table 3. Average Error Variance for the Mesoscale Part of Field

<table>
<thead>
<tr>
<th>Case</th>
<th>Error, ms</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
</tr>
<tr>
<td>60 rays</td>
<td></td>
</tr>
<tr>
<td>Raw</td>
<td>61</td>
</tr>
<tr>
<td>Tracked</td>
<td>32</td>
</tr>
<tr>
<td>96 rays</td>
<td></td>
</tr>
<tr>
<td>Raw</td>
<td>51</td>
</tr>
<tr>
<td>Tracked</td>
<td>28</td>
</tr>
</tbody>
</table>

Average error variance is in percent

### Table 4. Average Error Variance for the Spatial Mean

<table>
<thead>
<tr>
<th>Case</th>
<th>Error, ms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
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<tr>
<td>60 rays</td>
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<td>Raw</td>
<td>34</td>
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<tr>
<td>Tracked</td>
<td>10</td>
</tr>
<tr>
<td>96 rays</td>
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<tr>
<td>Raw</td>
<td>19</td>
</tr>
<tr>
<td>Tracked</td>
<td>9</td>
</tr>
</tbody>
</table>

Average error variance is in percent.
position uncertainty. The uncertainties of the relative position estimates are much lower (about ±3 m in the best case). An absolute position error of 200 m is not a problem for the moored application of 1981, but if the instruments were free-drifting this uncertainty would accumulate.

Finally, the 1981 data were used to compare mooring tracking by bottom transponders to tracking by tomographic rays. Four days (103, 106, 109, 112) were selected as having the largest displacements of any period with working tracking, and the tracking position estimates (J. Spiesberger, personal communication, 1981) were compared to those by the tomography (Figure 10). Although the tracking data are shown bihourly, only four daily averaged tomographic estimates are shown. The mooring tracking has an arbitrary zero, and so the two can only be compared to within a constant offset (which has been roughly estimated in producing Figure 10). The large error bars include the absolute position uncertainty left from the anchor position errors, while the relative position errors are about 30-m rms. The errors seen in Figure 10 for large displacements may be due to bias in the inverse (a consequence of the least squares formalism) and to a possible rotation of the coordinate system of the mooring tracking transponders.

6. DISCUSSION

The error levels shown were picked with knowledge of the travel time measurement variances as observed in the 1981 experiment and in the 1983 400-Hz tomography experiment [Worcester et al., 1985]. For a ray arrival well separated in time from other arrivals (sometimes impossible when narrow band equipment is used), most of the uncertainty in determining the pulse travel time is due to internal waves and is 3 – 5 ms for a single transmission at 200 – 400 Hz. If 24 arrivals are averaged and the measurement error is truly white (this error does not include mooring position or clock error), then the error is reduced to 1-ms rms. To achieve 0.2-ms rms, about 600 arrivals would have to be averaged. At present, the receivers normally store hourly data.

The dependence of performance on scale can be seen in the comparison between Tables 3 and 4 and in the inverse maps based on synthetic data. The spatial mean field and the 300-km waves are reproduced by most of the estimates, while the 150-km waves are poorly mapped until large numbers of rays and low error levels are reached. The tomographic array coupled with the stochastic inverse technique can be thought of as a system where the ocean is the input and the maps are the output. The system defined in this way resembles a low-pass filter, passing the large-scale variations without much change but distorting and attenuating the small scales. "Small" and "large" here have meaning relative to the spacing of ray coverage, which by reference to Figure 1 is between 25 and 75 km, consistent with the mesoscale-resolving experiment originally proposed by Munk and Wunsch [1979].

If the sound channel was not present, the rays would be

![Fig. 10. Plot of tomographic estimates of daily average mooring position (dots with error bars) and bihourly mooring tracking system estimates (crosses) for (a) east-west displacement of source 3, (b) north-south displacement of source 3, and (c) east-west displacement of source 1. The displacements are in meters, and the error bars include uncertainties in absolute position. The mooring tracking estimates are relative position only, and the origin has been shifted to superpose the two estimates. The mooring motion data set used here was worked up by J. Spiesberger.](image-url)
nearly straight lines, and the horizontal resolving power could be expressed in nondimensional form dependent on the array size. The sound channel complicates the analysis by permitting more rays per source/receiver pair as the range is increased, so that the minimum resolution scale will increase more slowly than a simple scaling argument would suggest.

For the 1981 data set, only about 60 medium quality rays were resolved, which even after averaging had rms errors of about 5 ms, so it is clear that tracking data are very important to the performance of the inverse (Figures 6, 7, 8, top left), and the decision to use the mooring tracking system was correct. If 400-Hz equipment [see Worcester et al., 1985] had been used, the situation would have been more like the 96-ray, 1-ms case, and the mooring tracking would have been less critical, provided only large-scale features were of interest.

A heuristic way to estimate the necessity of mooring tracking compares the number of unknown parameters with the number of independent data. The mooring and clock offsets represent four parameters per mooring, while if the rays between a single source-receiver pair are thought to contain no horizontal information, then there are effectively only as many parameters for the ocean as the product of the number of vertical modes times the number of source-receiver pairs. For the 1981 array, NS = 4, NR = 5, and three modes (NM = 3) were used, so about 60 parameters can be determined for the ocean. These represent the limiting mapping power of the array; the existence of this limit is suggested by the small improvements in mapping error corresponding to increases in the size of the corrected data set (Table 3). If absolute position, orientation, and clock setting are unimportant, then (60 + 36 - 4 =) 92 unknown parameters must be determined by the data. This would ideally require only 92 error-free, independent rays to approach the best resolution possible for the array. Real rays are neither error-free nor completely independent, and some parameters (horizontal position, first baroclinic mode amplitude) are more important (energetic) than others, but this rule of thumb allows a quick evaluation of an array. If all sources can reach all receivers with about NP independent paths per source-receiver pair, then NP · (NS · NR) data are available from the transmissions. Comparing NP · NS · NR to NS · NR · NM + (NS + NR - 1) · 4 shows that, provided NP > NM, as NS · NR become large, external mooring tracking becomes less necessary (and more expensive), so for large experiments, tracking would not be needed. If the vertical variations are complicated so that NM > NP, then mooring tracking adds independent information, but the estimation problem remains under-determined.

If free-drifting instruments (sources or receivers) were used in a tomographic experiment, the uncertainty in absolute position and orientation could grow over time. For this reason it would be necessary to fix the positions of two, preferably widely separated, instruments, either by satellite tracking or by using moored, towed, or shore-based instruments as part of the array. For free-drifting instruments the transmission schedule would also have to be often enough so that the drifters could not move so far between fixes as to exceed the linearity of the position-to-

travel-time forward model and/or radically alter the ray identification. The ability to predict new positions for tomographic instruments and floats based on old behavior would greatly ease this constraint.

Finally, the problem of linearity must be addressed, both in the ocean propagation approximation and in the plane wave approximation used to linearize the mooring position forward problem. (Clock drift is totally linear.) The linearity of the travel time perturbations due to sound speed changes for these ranges has been addressed by Spiesberger and Worcester [1983], who found that for sound speed perturbations of order 10 m s⁻¹ the linear travel times are in error by about 1 ms. As the range increases, the linearity approximation for the travel time becomes more sensitive.

During the 1981 experiment the sound speed perturbations were small, but currents were large enough to change some instrument depths by hundreds of meters. It is necessary to know when to expect travel time errors due to the breakdown of the approximation which led to equations (6)-(9). The second-order terms are good estimates of the nonlinearities, provided the displacements are small and can be used to estimate the limits on the linearization. For the rays used in this paper, nonlinear terms of about 1 ms occur for horizontal displacements of 1 km and vertical displacements of 100 m. The linearization is most sensitive to vertical displacements, and so it would be logical (and relatively simple) to put pressure sensors on acoustic instruments so as to supply the vertical motion and obviate the most troublesome part of the approximation. It is helpful to know the depths of the instruments to reasonable accuracy a priori for initializing the ray identification, and the vertical motion can also be used as an indication of horizontal motion, since the instrument must sink as the mooring leans. Because the expected travel time changes due to vertical excursions are two orders of magnitude smaller than the changes expected from horizontal motion, they are more difficult to recover. Finally, the independent measurement of vertical displacement enhances the use of the moving instruments as incoherent beam formers, so that each instrument can become a synthetic array.

The idea of using a moving instrument to make a synthetic array (as in synthetic aperture radar) may be extended to an "array" of drifting tomographic instruments as part of a monitoring experiment. The instruments may be tracked by the ocean mapping inverses, and if the absolute positions of two instruments are known, then the absolute positions of the entire array can be inferred. The motion of the drifters would add independent ray paths to the data set with each transmission, in contrast to fixed instruments. If the information from several transmissions could be retained using a model with data assimilation, the performance of the drifting array would outstrip a similar, but fixed, array. Finally, the motions of the drifters themselves would provide velocity information which could be added into the inverse/updating scheme.

Acknowledgments. This work was supported by a Mellon Foundation grant at Scripps Institution of Oceanography. The data came from the combined efforts of the Ocean Acoustic Tomography Group, in particular the experimental efforts headed by
Robert Spindel and Peter Worcester. Walter Munk, Carl Wunsch, and Bob Knox provided helpful discussion and suggestions, as did Peter Worcester, whose editorial advice was invaluable.

REFERENCES


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(Received February 25, 1985; accepted April 7, 1985.)