Variation of Residual Current in the Seto Inland Sea Driven by Sea Level Difference Between the Bungo and Kii Channels

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Abstract Understanding residual (i.e., total minus tidal) currents in coastal seas is important because the residual currents affect long-term material transports. In the Seto Inland Sea, Japan (SIS), a Bungo Channel to Kii Channel sea level difference causes a horizontal pressure gradient in the SIS and thus affects the residual current in the SIS. This study applies a linear regression method to examine how the residual current responds to the Bungo-Kii sea level difference. The residual current is obtained using the reciprocal acoustic transmission data collected in the eastern portion of the Aki-Nada sea area in 2012. The residual currents are estimated in the following three periods: from 12 April to 9 June, from 15 June to 21 July, and from 20 September to 27 October. In the regression analysis, an additional term is included to account for the fortnightly variation of the tide-induced residual current. More than 75% of the observed residual currents can be explained by the sea level difference and the fortnightly variation. For the three periods, the variations of the residual current along the acoustic transmission line are, respectively, 0.20 ± 0.01, 0.22 ± 0.01, and 0.30 ± 0.01 cm s⁻¹ per 1 cm of the Bungo-Kii sea level difference. The corresponding variations in the volume transport are 920 ± 149, 1,040± 170, and 1,390± 223 m³ s⁻¹ per 1 cm sea level difference. Comparing with the wind-induced volume transport, we find that the sea level difference can cause a comparable volume transport variation.

1. Introduction

Residual current, defined as the difference between the total current and linearly predicted tidal current, is a major topic in coastal oceanography. An important role of the residual current is transporting substances, for example, nutrients and pollutants. In the Seto Inland Sea, Japan (SIS; Figure 1), the residual current is a result of multiple processes such as the density-driven current, the wind-driven current, and the tide-induced residual current (nonlinear interactions of the oscillating tidal currents). A number of studies have been made to understand these processes. Some of the results are summarized in the review paper by Takeoka (2002).

Another process contributes to the residual current in the SIS. The SIS connects with the Pacific Ocean by two channels: the Bungo Channel on the west and the Kii Channel on the east (Figure 1b). The Kuroshio flowing south of Japan induces a time-dependent variation of the sea level height along the south coast of Japan (e.g., Zhang & Ichikawa, 2005). The Bungo-Kii sea level difference creates a horizontal pressure gradient in the SIS. This horizontal pressure gradient drives the throughflow in the SIS and affects the residual currents at each location throughout the SIS. In the SIS, the sea levels are recorded operationally at multiple tide gauge stations. Kunii and Fujiwara (2006) analyzed the multiple sea level data in the SIS and reported that the daily mean Bungo-Kii sea level difference is mainly accounted for by the sea level difference between the western and eastern portions of the Bisan Strait. They also examined the relationship between the Bungo-Kii sea level difference and the volume transport at the Bisan Strait using a numerical ocean model; the results suggest that a 1 cm increment in the Bungo-Kii sea level difference corresponds to an increase in a volume transport of 840 m³ s⁻¹. Komai et al. (2008) also used an ocean model and found that the variations of the volume transport per centimeter of sea level difference are 1,046 and 3,315 m³ s⁻¹, respectively, with and without accounting for the spatial variation of the density in the SIS.

The residual current and the volume transport in the SIS vary with time due to the temporal variability of Kuroshio (e.g., Kawabe, 1995; Miyama & Miyazawa, 2014). In the SIS, it is difficult to make long-term and/or
operational observations of the current velocity using moorings equipped with current meters or acoustic Doppler current profilers (ADCPs), although they are the widely accepted methods to measure the time variation of the current at a particular location. The difficulty partly results from heavy shipping and fishing activities. Also, since the SIS maintains high biological productivity, subsurface instruments require frequent maintenance due to biofouling. Measuring the volume transport may be particularly difficult; the spatial interpolation using an insufficient number of the point-measured velocity data would bias the estimates. In the SIS, practical methods to make an operational observation of the current velocity may be using HF-radars (adopted at the Osaka Bay and the Kii Channel) or the acoustic method described below.

A useful approach to obtain the current measurements in inland seas is measuring travel times of acoustic pulses transmitted reciprocally between horizontally separated acoustic transceivers. The idea behind the method is based upon the fact that the time for sound traveling along an acoustic ray path in the ocean is a function of the sound speed, which is primarily a function of temperature, and current velocity; sound travels faster in warm water than in cold water, and along a current than against it. The effect of sound speed and current on the travel times can be separated by making reciprocal transmissions; the current magnitude is related to the differential travel time of the reciprocal transmission (Howe et al., 1987; Munk et al., 1995; Worcester, 1977). The reciprocal acoustic transmissions may be particularly suitable for measuring the volume transport due to its integral (i.e., path-average) nature. The method has been used to measure the volume transport through straits (e.g., Send et al., 2002; Zhu et al., 2015). Since the acoustic transceivers are deployed from quays or near shores, the travel time measurements may not be interrupted by fishing and shipping activities. Deploying the transceivers from the quays also makes system maintenance easy and enables long-term operational observations.

The travel time method has been applied to current measurements in the SIS by a research group of Hiroshima University (Adityawarman et al., 2011; Lin et al., 2005; Park & Kaneko, 2000; Yamaguchi et al., 2005; Yamoaka et al., 2002; Zheng et al., 1997). Those applications in coastal seas are referred to as coastal acoustic tomography (CAT). In 2012, a CAT experiment was conducted in the eastern portion of the Aki-Nada sea area (nada in the SIS indicates relatively wide basins connected by relatively narrow straits) with the aim of
estimating the volume transport through the SIS (Figure 1c). Zhang et al. (2016) analyzed the reciprocal travel time data over 6 months and concluded that the net westward transport averaged monthly for the successfully observed period was $13,100 \pm 2,500 \text{ m}^3 \text{s}^{-1}$. In the experiment, the reciprocal acoustic transmissions were conducted along one transmission line (Figure 1c). To estimate the net volume transport, the representative angle of the flow was determined by comparing the observed path-averaged current with the current predicted from the sea level elevation (Zhang et al., 2016).

The Bungo-Kii sea level difference is one of the forces that drive the throughflow (and the residual current) in the SIS. To understand the physical processes in the SIS, it is important to estimate the relationship between the sea level difference and the variation of the residual current. Therefore, in this study, we analyzed the acoustic data obtained from the 2012 CAT experiment but examined how the residual current at the observation site responds to the Bungo-Kii sea level difference. Specifically, a linear regression method was used to estimate the variations of the residual current and the volume transport as a function of the Bungo-Kii sea level difference. The sea level at each channel was determined using the mean of the tide gauge data from two stations located at both sides of each channel. To estimate the variation of the volume transport, we determined the representative angle of the residual current following Zhang et al. (2016).

Note that this study only focuses on the relationship between the sea level difference and the residual current even though other factors such as the spatial variation of density, the wind stress, and the bottom friction also contribute to the SIS throughflow (and the residual current at the observation site).

The paper is organized as follows. Section 2 describes the data analysis method and the estimation of the variation of the residual current and the volume transport as a function of the Bungo-Kii sea level difference. The results and discussion are presented in sections 3 and 4, respectively. Finally, concluding remarks are made in section 5, including a description of a future CAT experiment in the SIS.

2. Data and Methods

2.1. Residual Currents Obtained From Acoustic Travel Time Method

The residual current was derived from the reciprocal transmission experiment conducted at Aki-Nada in 2012 (Zhang et al., 2016). During the experiment, two CAT systems (T1 and T2 in Figure 1c) repeated acoustic transmissions every 10 min. The distance between the two acoustic transceivers was 13,769 m. Zhang et al. (2016) provide the details of the experiment. The acoustic transmission experiment was intermittent due to occasional system malfunctions. We focused on the residual current for three periods separately: from 12 April to 9 June (referred to as Period I), from 15 June to 21 July (Period II), and from 20 September to 27 October (Period III). Within each period, continual data gaps were less than 4 h. The continual data gaps of a few hours within each period do not affect the estimates of the relationship between the residual current and the sea level difference.

The data obtained from the CAT experiment were the arrival patterns of the acoustic pulses (see Figure 2 in section 3). In the experiment, due to the finite duration of the source pulse, the acoustic rays traveling along different paths formed a broad arrival pulse. Since this broad pulse results from the interference of the refracted-surface-reflected rays and the refracted rays, its peak time cannot represent the travel times for the similar path rays. The number of those arrival pulses and their magnitude in the arrival patterns changed considerably with time. Due to the propagation characteristics in this environment, we found it difficult to determine the reciprocal travel times using the maximum of the largest arrival peak. Instead, we define the travel time as the time when the magnitude of arrival pulse rises to 14 dB. For most of the observed arrival patterns in the experiment, this definition determines the travel time of the first arrival ray (see Figure 2) but underestimates the value slightly. The underestimated parts cancel out in calculating the difference of the reciprocal travel times. When the peak magnitude of the arrival patterns was less than 14 dB, we decided that those data were unusable. In the observation site, strong tidal currents cause vertical mixing and weaken thermal stratification (e.g., Takeoka, 2002; Yanagi & Okada, 1993). During the experimental periods, the sound speed increased with depth except occasionally for the upper few meters (Zhang et al., 2016). When the sound speed increases with depth, the first arrival ray is the refracted-surface-reflected ray with the turning depth near the seafloor. The first arrival ray was the one that has surface reflections and lower turning depths ranging from about 20 to 25 m seasonally (Zhang et al., 2016). Using...
different arrival ray data would result in inaccurate estimates of the residual current or variation of volume transport.

From the travel times determined in reciprocal directions ($\tau_1$ and $\tau_2$ for those received at T1 and T2, respectively), we estimated the path-averaged sound speed $c$ and the path-averaged current $v$, which is the projection of the true current onto the line T2–T1, using the following equations (Zheng et al., 1997):

$$c = \frac{R}{\tau_m},$$

$$v = -\frac{c^2}{2R} \tau_d,$$

where $\tau_m = (\tau_1 + \tau_2)/2$ and $\tau_d = \tau_1 - \tau_2$ are the mean and the difference of the reciprocal travel times, respectively. $R$ is the distance between the transceivers. Positive $v$ (i.e., $\tau_d < 0$) indicates that the projected current is directed toward the T1 station and corresponds to water movements from Aki-Nada to Hiuchi-Nada (i.e., from west to east) at the observation site.

The estimated path-averaged current $v$ was occasionally implausible due to the error in the selected $\tau_1$ and/or $\tau_2$. We applied a harmonic analysis to the time series of $v$ using the UTide package written in MatLab (Codiga, 2011); when the difference between the estimated $v$ and the harmonically predicted $v$ was larger than 0.2 m s$^{-1}$, we decided that the estimated $v$ was unusable. This threshold (0.2 m s$^{-1}$) was determined by comparing the estimated and the harmonically predicted values. Its difference depends on several factors: measurement noise, the number of constituents used in the harmonic analysis, nontidal current variations such as wind-induced and density-induced currents and the current due to the Bungo-Kii sea level difference. In the harmonic analysis, we considered the N$_2$ constituent in addition to the four major
constituents, namely, M2, S2, K1, and O1 constituents. Semidiurnal variations dominated the estimated \( v \). The amplitude of the N2 constituent was larger than those of the K1 and O1 constituents, and the N2 constituent was necessary for precise prediction of \( v \). Since the data lengths of all three periods were longer than 28 days, the harmonic analysis could separate the contribution from the M2 and N2 constituents in all three periods. The data gaps (due to the system malfunctions less than 4 h, low arrival magnitude, and implausible \( v \)) were linearly interpolated.

In this study, we consider the daily mean of the observed \( v \) to be the residual current. To ensure that the major diurnal and semidiurnal signals do not affect the estimate of the residual current, the variations due to the M2, S2, K1, and N2 constituents were first removed from \( v \) using the result of the harmonic analysis. Then, the other diurnal and shorter-period signals and also the measurement noise in \( v \) were attenuated by a moving average (Gustafsson, 1996) with a 151-point (\( \sim 25 \) h) rectangular window. This moving average was also applied to the result of the harmonic analysis, yielding a negligible error resulting from the harmonic analysis. Finally, the filtered data were down-sampled every 24 h. These filtered and down-sampled data, denoted by \( \tilde{v} \), are the residual currents (along the transmission line).

### 2.2. Estimation of Sea Level Difference

The sea level difference between the Bungo and Kii Channels was estimated using the hourly sea level records at four tide gauge stations: Uwajima and Saiki for the Bungo Channel and Wakayama and Komatsu-shima for the Kii Channel (Figure 1b). The reference of the sea level was the mean sea level measured at Tokyo Bay. To construct the daily sea level time series, the hourly sea level data were processed in a way similar to the current data, with a 25-point (25 h) rectangular window. The daily sea level difference was the difference between the mean sea levels of these two channels (\( \Delta \eta = \eta_{\text{Bungo}} - \eta_{\text{Kii}} \)). Positive \( \Delta \eta \) indicates that the sea level in the Bungo Channel was higher than that in the Kii Channel; simplistically then, for such a situation, flow would be positive to the east within the SIS and through Aki-Nada.

Regional differences in the atmospheric pressure at the sea surface (\( P_a \)) induce water movements. In this study, \( P_a \) was taken into account by correcting the sea levels; assuming that a pressure of 1 hPa corresponds to a sea level variation of 1 cm, we added (\( P_a - 1.013 \)) (cm) to the sea level data. For this processing, the daily mean \( P_a \) data at the meteorological stations near the tide stations were downloaded from the website of Japan Meteorological Agency. For all three periods, the root mean square (RMS) difference of (\( P_a - 1.013 \)) observed between two channels was about 1.1 cm and this value corresponded to about 4% of the variance of Bungo-Kii sea level difference. Hereinafter, \( \Delta \eta \) includes the atmospheric pressure correction.

### 2.3. Regression Analysis on the Sea Level Difference and the Residual Current

A linear regression method was applied to \( \tilde{v} \) and \( \Delta \eta \) focusing on the deviation from the mean: \( \tilde{v}' = \tilde{v} - \langle \tilde{v} \rangle \) and \( \Delta \eta' = \Delta \eta - \langle \Delta \eta \rangle \), where the operator \( \langle \cdot \rangle \) indicates the time average over the experimental period. Note that this study does not focus on the mean currents \( \langle \tilde{v} \rangle \); due to geographic and bathymetric effects, the direction difference between the maximum flood and the maximum ebb currents may be different from 180°, causing an apparent mean current difference when \( \tilde{v} \) is averaged. The observed \( \tilde{v}' \) would have a fortnightly variation of the tide-induced residual current, corresponding to the MS2 tidal current constituent which results from the nonlinear interaction between the M2 and S2 tidal currents (Guo et al., 2013). Note that there may also be the contribution from \( \Delta \eta' \) to the fortnightly variation in \( \tilde{v}' \). Therefore, the following linear model was used to relate the time series of \( \tilde{v}'(n) \) to \( \Delta \eta'(n) \):

\[
\tilde{v}'(n) = a_0 \Delta \eta'(n) + a_2 \cos(2\pi n/T) + a_3 \sin(2\pi n/T) + e(n),
\]

where \( n \) is the index of the day and \( T (= 14.77 \text{ days}) \) is the period of the fortnightly variation (spring-neap tidal cycles). Although other constituent pairs may cause additional fortnightly and longer-period variations, this study takes only into account the variation with a period of 14.77 days (the MS2 constituent) since the M2 and S2 are the largest two tidal current constituents in the SIS. The time lag between the variations of \( \tilde{v}' \) and \( \Delta \eta' \) was ignored since the residual current should respond to the sea level difference within 1 day. The error term \( e(n) \) may account for not only the measurement noise but also the other components of the residual current such as density-driven and wind-driven currents.

The regression coefficients \( (a_1, a_2, \text{ and } a_3) \) were solved by the ordinary least squares method. A measure of the goodness of fit of the regression curves (equation (3)) may be given by the coefficient of determination:
where $\hat{v}'$ is the reconstruction of $v'$ using equation (3) with the estimated coefficients ($\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$). Note that obtained $R^2$ value equals the square of the correlation coefficient between $v'$ and $\hat{v}'$.

### 2.4. Estimation of Volume Transport Variation

The regression coefficient $a_1$ corresponds to the variation of the residual current (along the T2–T1 line) per 1 cm sea level difference. From $a_1$, we estimated the corresponding coefficient for volume transport variation. Assuming that the observed path-averaged current is equivalent to the section-averaged (i.e., the depth-averaged and range-averaged) current, the coefficient for the volume transport variation, $a_p$, was estimated by

$$a_p = a_1 \tan \theta,$$

where $A$ is the cross-sectional area from the sea surface to the sea floor along the line T2–T1, and $\theta$ is the angle between the line T2–T1 and the representative direction of the residual current. The area $A$ was estimated to be 513,910 m$^2$ using 500 m mesh bathymetry data (J-EGG500 from Japan Oceanographic Data Center). We did not account for Stokes transport, which may be expressed as the product of the instantaneous sea surface elevation and velocity (e.g., Uncles et al., 1985). The effect of the Stokes transport on the total volume transport was neglected here since the sea level variation is small compared with the mean water depth.

Since one cannot obtain $\theta$ directly from the reciprocal transmissions along one transmission line, we followed the procedure described in Zhang et al. (2016) to estimate $\theta$ (described below). This study considers only the $M_2$ constituent, which accounts for about 80% of the variance in the observed path-averaged current $v$, while Zhang et al. (2016) considered the four major constituents. First, we predicted the sea level variation of the $M_2$ constituent ($\eta_p$) at Kikuma (Figure 1c) according to the Tidal Harmonic Constants Tables issued by the Hydrographic Department, Japan Maritime Safety Agency (1992). Then, the current at Kikuma was predicted ($v_h$) by the following equation, which is obtained from the linear equations for a shallow water wave traveling in an idealized ocean of flat bottom without friction:

$$v_p = \eta_p \sqrt{g/H},$$

where $g$ (9.8 m s$^{-2}$) is the acceleration due to gravity. The water depth $H$ was estimated using the mean water depth over the area indicated by the dashed box in Figure 1c and has a value of 36 m using the J-EGG500 bathymetry data. The predicted current $v_p$ was considered as the magnitude of the true current velocity. Next, the observed $M_2$ tidal current ($v_n$) was extracted via the harmonic analysis from the observed path-averaged current $v$. Then, from the relationship

$$v_n = v_p \cos \theta,$$

the representative angle $\theta$ was determined via a least squares method. Note that the tidal signals at the observation site are not perfectly progressive waves; $v_n$ was time-shifted 1.17 h prior to the least squares method (Zhang et al., 2016). The estimated representative angle $\theta$ and its uncertainty $\Delta \theta$ were obtained from the mean and the standard deviation of the $\theta$ values estimated from the three individual periods.

Assuming the uncertainties of $a_1$, $A$, and $\theta$ are statistically independent, the uncertainty of the volume transport variation is estimated by

$$\Delta(a_1 \tan \theta) = \sqrt{(A \tan \theta \Delta a_1)^2 + (a_1 \tan \theta \Delta A)^2 + (a_1 \tan \theta \Delta \theta)^2},$$

where $\Delta A$ (910 m$^2$) is obtained by the difference in the estimate of $A$ between this study (513,910 m$^2$) and Zhang et al. (2016) (477,936 m$^2$). $\Delta a_1$ is obtained using an uncertainty analysis of the regression coefficients (see Appendix A).
3. Results

Figure 2 shows the examples of the reciprocal arrival pattern obtained from two transmissions: (a) 10 April, 12:00 and (b) 4 October, 11:10. The difference in the horizontal axis (the time delay from the start of the acoustic transmission) between (a) and (b) is due to a change of the ocean sound speed (mainly from temperature) at these two times. The acoustic pulses traveled faster at (b) with a mean sound speed of 1,532 m s\(^{-1}\) than at (a) of 1,493 m s\(^{-1}\). The reciprocal arrival pattern in (a) shows a simpler structure than that in (b). In (b), the estimated differential travel time would be problematic if the travel time was determined from the highest peak arrival. Using the travel time defined here, which corresponds to the time when the magnitude of the acoustic pulse rises to a threshold (14 dB; the dashed line in Figure 2), the number of erroneous v's could be reduced.

Figure 3 shows time series of (a) the hourly sea level at Matsuyama (located at about 12 km west from Kikuma; black triangular symbol in Figure 1c) and (b) the path-averaged current \(v\) and sound speed \(c\) observed from the acoustic measurements. The observation time is from 18 April to 26 April of Period I. The positive value of \(v\) corresponds to the water movement from Aki-Nada to Hiuchi-Nada. From the harmonic analysis, we found that the semiidiurnal signals observed in the tidal current (from the acoustic measurements) are more dominant than those in the sea level height (from the tide gauge data at Matsuyama). During Period I, the tidal current amplitudes of the \(M_2, S_2, N_2, K_1,\) and \(O_1\) constituents were 41.8, 14.3, 8.7, 7.7, and 3.7 cm s\(^{-1}\), respectively, while the tidal amplitudes of these constituents at Matsuyama are 99.3, 40.8, 18.0, 31.0, and 22.7 cm (Hydrographic Department, Japan Maritime Safety Agency, 1992). As seen in Figure 3, diurnal inequality in the path-averaged current (black dot in the bottom plot) is not as clear as that in the hourly sea level (top plot).

The observed path-averaged sound speed (gray line in Figure 3b) shows a semiidiurnal variation in addition to an increasing trend due to an increase in the sea temperature. The local minima and maxima of the sound speed appeared at the times almost coincident with slack waters (weak current speed). The sound speed around the time of slack water indicates whether Aki-Nada or Hiuchi-Nada has higher temperature. Before 21 April, the sound speed reached the local maximum around the end of eastward current (e.g., at 00:00 of 20 April), indicating that the temperature in Aki-Nada was higher than that in Hiuchi-Nada. After 23 April, the observed sound speed reached the local maximum at the end of the westward current (e.g., at 20:00 of 24 April), indicating that the temperature in Hiuchi-Nada was higher than that in Aki-Nada.

Figure 3. Time series of (a) the sea level at Matsuyama and (b) the observed path-averaged current \(v\) (black dot) and the sound speed \(c\) (gray line) from 18 to 26 April.
Table 1 summarizes the results of the path-averaged current $v$ estimated using the data from individual periods. The path-averaged current $v$ was successfully estimated most of the times during Period I; the number of unusable data was only 3% of the total number of data. During Periods II and III, the number of unusable data increased to 10% and 21%, respectively. Note that if we determined $v$ using the travel times of the highest arrival peak (Zhang et al., 2016), the numbers of unusable data would increase to 21%, 37%, and 31% for Periods I, II, and III, respectively, due to an increase in the number of implausible $v$. However, it seems that Zhang et al. (2016) did not remove those implausible data from the analysis. As a result, our travel time definition resulted in a smaller $r_v$ (the RMS deviation between $v$ and its moving average; see Appendix A) compared with theirs. That is, while $r_v$ in Zhang et al. (2016) was about 11.6 cm s$^{-1}$, $r_v$'s in this study were 2.7, 4.3, and 2.8 cm s$^{-1}$ for Periods I, II, and III, respectively.

The representative angle $\theta$, from independent estimates of three individual periods (Table 1), was $42^\circ \pm 4^\circ$, and this value was used to estimate the volume transport variation and its uncertainty via equations (5) and (8).

Figure 4 shows time series of (a) the sea level height at Matsuyama, (b) the sea level difference between the Bungo and Kii Channels ($D$), and (c) the variation of the residual current ($v'$) during Period I. The right axis in Figure 4c indicates the corresponding volume transport variation ($Q'$). The fortnightly variation of $v'$ is also found; the westward current (negative $v'$) is intensified at the spring tide.

The results of the regression analysis are summarized in Table 2 and are presented as a time series in Figure 5. Plots a, c, and e of Figure 5 show the time series of the observed residual current $v'$ (solid line) and the reconstructed residual current $\hat{v}'$ (dashed line). Plots b, d, and f of Figure 5 show the terms from the right-hand side of equation (3), with the solid line showing $a_1 \Delta y'(n)$ and the dashed line showing $a_2 \cos(2\pi n/T) + a_3 \sin(2\pi n/T)$, in which the coefficients were estimated using the data from each period. The sum of these two variations shown in plots b, d, and f is the reconstructed residual current shown in plots a, c, and e, respectively.

Table 1

<table>
<thead>
<tr>
<th>Period</th>
<th>Usable data (%)</th>
<th>$\sigma_v$, $\sigma_{v'}$ (cm s$^{-1}$)</th>
<th>$\theta$ (°)</th>
<th>$\langle v'^2 \rangle^{1/2}$ (cm s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>97</td>
<td>2.7, 0.22</td>
<td>38.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Period II</td>
<td>90</td>
<td>4.3, 0.35</td>
<td>45.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Period III</td>
<td>79</td>
<td>2.8, 0.23</td>
<td>41.1</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Note. See Appendix A for the error in $v$ and $v'$ ($\sigma_v$ and $\sigma_{v'}$).

Figure 4. Time series of (a) the hourly sea level ($\eta$) at Matsuyama, (b) the sea level difference between the Bungo and Kii Channels ($\Delta y'$), and (c) the variation of the residual current ($v'$) during Period I. The right axis in Figure 4c indicates the corresponding volume transport variation ($Q'$).
reconstructed residual current reproduced the typical variation observed in the residual current. The values of $R^2$ were 0.76, 0.77, and 0.80 for Periods I, II, and III, respectively (Table 2), i.e., more than 75% of the $v'$ variance is explained by the sea level difference and the fortnightly variation.

The values of $\hat{a}_1$ and $\sqrt{a_1^2 + a_2^2}$ for Period I were almost the same as those for Period II (Table 2). For Period I, the current variations due to the sea level difference, $\hat{a}_1\Delta v'(n)$, and the fortnightly variation, $a_2\cos(2\pi n/T) + a_3\sin(2\pi n/T)$, had similar amplitudes (Figure 5b). These two time series are relatively in phase in the middle of April and in May but are out of phase at the end of April and at the beginning of

**Table 2**

<table>
<thead>
<tr>
<th></th>
<th>$\hat{a}_1$</th>
<th>$\sqrt{a_1^2 + a_2^2}$</th>
<th>$\Delta a_1$</th>
<th>$(\Delta a_1^2 + \Delta a_2^2)^{1/2}$</th>
<th>MSE (cm$^2$ s$^{-2}$)</th>
<th>MV (cm$^2$ s$^{-2}$)</th>
<th>$R^2 = 1 - \frac{MSE}{MV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>0.20</td>
<td>1.06</td>
<td>0.01</td>
<td>0.06</td>
<td>0.47</td>
<td>1.99</td>
<td>0.76</td>
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<tr>
<td>Period II</td>
<td>0.22</td>
<td>1.02</td>
<td>0.01</td>
<td>0.13</td>
<td>0.56</td>
<td>2.46</td>
<td>0.77</td>
</tr>
<tr>
<td>Period III</td>
<td>0.30</td>
<td>1.98</td>
<td>0.01</td>
<td>0.13</td>
<td>1.50</td>
<td>7.40</td>
<td>0.80</td>
</tr>
<tr>
<td>All</td>
<td>0.24</td>
<td>1.32</td>
<td>0.01</td>
<td>0.05</td>
<td>1.22</td>
<td>3.95</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Figure 5. Results of the linear regression for the Periods I, II, and III data. For each period, (a), (c), and (e) comparison of the observed residual current $v'$ (solid line) and the reconstructed $\hat{v}'$ (dashed) using the data from the period as well as the reconstructed result (dotted) using all three periods. (b), (d), and (f) contribution of the residual current from the sea level difference (solid; the first term on the right-hand side of equation (3)) and the fortnightly variation (dashed; the sum of the second and third terms).
Table 3

<table>
<thead>
<tr>
<th>Period</th>
<th>$\Delta a_1$ (1 cm sea level difference)</th>
<th>$\sigma_1$ (1 cm sea level difference)</th>
<th>$\sigma_2$ (1 cm sea level difference)</th>
<th>$\sigma_3$ (1 cm sea level difference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>36</td>
<td>65</td>
<td>129</td>
<td>920 ± 149</td>
</tr>
<tr>
<td>II</td>
<td>50</td>
<td>73</td>
<td>145</td>
<td>1,040 ± 170</td>
</tr>
<tr>
<td>III</td>
<td>50</td>
<td>97</td>
<td>194</td>
<td>1,390 ± 223</td>
</tr>
<tr>
<td>All</td>
<td>21</td>
<td>78</td>
<td>156</td>
<td>1,120 ± 176</td>
</tr>
</tbody>
</table>

*Note: Unit: m$^3$ s$^{-1}$ per 1 cm sea level difference.*

For Period III, the observed residual current ($\dot{v}'$) (solid line in Figure 5e) shows an increasing trend in addition to the fortnightly variation. Although the reconstructed residual current ($\hat{v}'$) (gray dashed line in Figure 5e) reproduced the general trend, the consistency of the shorter-period variation between $\dot{v}'$ and $\hat{v}'$ is not as good as those for Periods I and II. The mean of the squared reconstruction errors (MSE; $(\langle\dot{v}' - \hat{v}'\rangle^2)$) of Period III is larger than those of Periods I and II (Table 2). Despite the larger MSE, $R^2$ for Period III is slightly higher than those for Periods I and II since during Period III the mean variance in the observed residual current (MV; $\langle v'^2\rangle$) is also higher (the square-root values are also listed in the last column of Table 1). For the Period III data, both $\hat{a}_1$ and $\sqrt{a_1^2 + a_3^2}$ are larger than those for the Periods I and II data (the difference is larger than the uncertainty), implying a seasonal variability.

The regression analysis was also performed using all 6 month data. The estimated values of the regression coefficients were close to the mean of three estimates from individual periods (Table 2). The amplitude of the reconstructed time series using all three periods is slightly larger than those of Periods I and II but is smaller than that of Period III (from the comparison of the dotted line and dashed line in Figures 5a, 5c, and 5e). The $R^2$ value decreased to 0.69, indicating that for the entire 6 month data the present linear regression model might not be applicable and multivariate regression might be more appropriate.

The variation of the volume transport due to the 1 cm sea level difference between the Bungo and Kii Channels was estimated via equation (5). Table 3 lists, from left to right columns, the contribution of the uncertainties of $a_1$, $A$, and $\theta$ to the estimate of the volume transport variation. During Periods I, II, and III, the estimated variations of the volume transport and their uncertainties ($a_q$) respectively, were $920 \pm 149$, $1,040 \pm 170$, and $1,390 \pm 223$ m$^3$ s$^{-1}$ per 1 cm sea level difference. In the present study, $a_q$ mainly resulted from the uncertainty of the representative angle ($\Delta \theta$).

4. Discussion

We have examined how the SIS residual current responds to the Bungo-Kii sea level difference by applying the linear regression method to both the sea level difference and the residual current obtained from the reciprocal acoustic transmission data of the CAT experiment. In addition to the sea level difference, the fortnightly variation was taken into account to explain the tide-induced residual current. These two factors account for more than 75% of the variance of the residual current at the observation site. The remaining variance might be due to the error in the residual current estimate and to additional physical processes not included in the analysis. The estimates of $a_1$ using the data of Periods I, II, and III indicates that a 1 cm increment of the Bungo-Kii sea level difference increases the residual currents by $0.20 \pm 0.01$, $0.22 \pm 0.01$, and $0.30 \pm 0.01$ cm s$^{-1}$ along the transmission line, respectively.

Due to the complicated acoustic arrival patterns commonly observed at the observation site, the estimation of the travel time is difficult, especially for estimating the residual currents, which show much smaller variation compared with the tidal current. For these three periods, the travel time defined in Zhang et al. (2016) yielded different estimates of the residual current variation: $0.25 \pm 0.03$, $0.26 \pm 0.03$, and $0.13 \pm 0.03$ cm s$^{-1}$ per 1 cm sea level difference. Using their estimated residual currents, the regression analysis explained 47%, 45%, and 39% (100$R^2$) of the total variance in the residual current. Compared with the results using
our travel time definition, their relatively low $R^2$ values may be due to larger error in the estimated travel times.

As the acoustic transmissions were conducted along one transmission line (Figure 1c), the representative angle of the residual current to the transmission line was estimated following Zhang et al. (2016) to obtain the variation of the volume transport. The volume transport variations due to 1 cm sea level difference ($\Delta h$) were $920 \pm 149$, $1,040 \pm 170$, and $1,390 \pm 223$ m$^3$ s$^{-1}$ for Periods I, II, and III, respectively. Our estimates substantiate the results from previous numerical ocean model studies (Komai et al., 2008; Kunii & Fujiwara, 2006). The percentage uncertainty in the volume transport variation is about 16%, mainly due to the uncertainty in the representative angle of the residual current, $\Delta \theta$. Approximating the section-averaged current from the observed path-averaged current might introduce an additional uncertainty in estimating volume transport since the acoustic ray paths do not sample the whole water column uniformly. Furthermore, we did not consider the volume transport through the northern channel (Figure 1c).

The observed residual current might include the variation due to the spatial variability of the density in the SIS, i.e., density-driven flow. In the SIS, the density-driven flow develops during the summer and at the observations site may be westward near the surface and eastward near the bottom (e.g., Guo et al., 2004). It is possible that this westward density flow in the surface layer weakened the response of the residual current to the sea level difference for Periods I and II compared with that for Period III. Komai et al. (2008) simulated the SIS volume transport with and without the spatial variation of the density in the SIS, and they showed that the volume transport per 1 cm sea level difference was larger for the uniform density case. Although the variation of the density-driven flow was not explicitly accounted for in the regression analysis, some of its variation might be explained by the term of the fortnightly variation. This is because the density fields vary during spring-neap tidal cycles both in the SIS and in the Bungo and Kii Channels (e.g., Nagai & Hibiya, 2012; Takeoka et al., 1993; Yu et al., 2016). An example of the fortnightly variation of the residual current due to the variation of the density-driven current is observed in Tokyo Bay (Yanagi et al., 2003). Further studies are required to incorporate the effect of inhomogeneous density field on the residual current appropriately.

Winds were not considered in this study because the wind in Kurushima Strait was usually weak during the study period from April to October. In the SIS, winds play an important role in the mean current, particularly, in winter (e.g., Fujiwara et al., 1990; Fujiwara & Higo, 1986). Fujiwara and Higo (1986) suggested that the westerly wind drives an eastward throughflow of 4,000 m$^3$ s$^{-1}$ on average during winter. This volume transport induced by the wind is equivalent to the volume transport induced by sea level differences of about 4.3, 3.8, and 2.9 cm considering the estimated volume transport variation per 1 cm sea level difference (the values obtained by $4,000/\Delta h$) during Periods I, II, and III, respectively. These values can be found in $\Delta \theta$ and also in monthly means of the Bungo-Kii sea level difference (e.g., Kunii & Fujiwara, 2006), indicating that the sea level difference can cause a volume transport variation comparable with that induced by winds (at least during the periods with weak winds). Successful reciprocal acoustic measurements during the winter can be used to explore the contributions of the winds and sea level difference to the SIS throughflow.

5. Concluding Remarks

The reciprocal acoustic measurement is a useful approach to measuring the residual current in the SIS. The residual current in the SIS can be investigated in further detail with a more sophisticated acoustic transmission experiment. For example, conducting the reciprocal transmissions between multiple transceiver pairs enables us to estimate both the current magnitude and direction, resulting in a smaller uncertainty in estimating the volume transport. Measuring both the magnitude and direction would also improve the estimates of the net volume transport and the resulting residence time of water in the SIS (Zhang et al., 2016). When more transceivers are available, the spatial distribution of residual currents can be obtained via solving an inverse problem (e.g., Zhu et al., 2017). Using a data assimilation scheme, one may estimate the spatial distribution of residual currents even with a pair of the transceivers. The spatial variation in the SIS residual current can be resolved by a better spatial coverage of the acoustic transmission lines. With years-long successful transmissions, we will be able to determine the seasonal variation and the dependence on the Kuroshio. The accurate knowledge of the acoustic propagation is also necessary since the estimated current is the averaged value along the acoustic path while we need the section-averaged current. The
observed sound speed may also contribute to estimate of the density variation in the observation area. These improvements of the method are useful not only for determining the precise residual current but also for validating numerical ocean models, which are inevitably required to fully understand the full current system in the Seto Inland Sea.

Appendix A: Error Estimation of the Regression Coefficients

Equation (3) can be written in the following matrix form:

\[ \mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{e} \]  

(A1)

where \( \mathbf{d} \) and \( \mathbf{e} \) correspond to the time series of \( \nu^i(n) \) and \( e(n) \), respectively, and \( \mathbf{m} \) contains the coefficients \( (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3) \). The columns of the matrix \( \mathbf{G} \) contain, from left to right, \( \Delta\nu^i(n), \cos(2\pi n/T), \) and \( \sin(2\pi n/T) \), respectively. The ordinary least squares solution is given by

\[ \mathbf{m} = (\mathbf{G}^\mathsf{T}\mathbf{G})^{-1}\mathbf{G}^\mathsf{T}\mathbf{d}. \]  

(A2)

where \( \mathbf{G}^\mathsf{T} \) is the transpose of \( \mathbf{G} \). The estimated coefficient \( \hat{\mathbf{a}}_i \), i.e., \( \hat{\mathbf{m}}(1) \), corresponds to the residual current variation per 1 cm sea level difference.

Assuming that the error in the residual current is \( \sigma_e \) the variance of the ith regression coefficient is

\[ \Delta\hat{\mathbf{a}}_i = \sigma_e^2 \left( (\mathbf{G}^\mathsf{T}\mathbf{G})^{-1} \right)_{ii}. \]  

(A3)

To estimate \( \sigma_v \), we first calculated the RMS difference between \( \nu \) and its seven-point moving average; this RMS difference was considered as the error in \( \nu \) and denoted by \( \sigma_v \). Then, \( \sigma_v \) was given by \( \sigma_v / \sqrt{15T} \).

References


