I. INTRODUCTION

Acousticians have used incoherent processing of ambient noise to achieve small scale imaging in the ocean\textsuperscript{1} in the same way as optical imaging is performed from incoherent light. Here, we show that it is feasible to transform a collection of apparently incoherent noise sources into a coherent, large scale, imaging field. Actually, every individual source of noise (e.g., a collapsing bubble) in the ocean generates an acoustic field that is potentially coherent when received between two points after long range propagation. However, this small coherent component at each receiver point is buried in the spatially and temporally incoherent field produced by all the widespread noise sources distributed over the ocean. We demonstrate in this paper that a long-time cross-correlation process extracts coherent wave fronts from ambient noise without the support of any identifiable source. This means that noise could be used as a potential coherent source in the ocean leading to the concept of a self-imaging process. These general results reveal the potential information content of random noise fields in a natural environment.

The sources of ocean surface noise\textsuperscript{2–4} (natural and man-made) as well as the subsequent average spatial distribution of ocean noise\textsuperscript{5–7} have been studied extensively. However, because the instantaneous distribution of all the mutually incoherent sources is extremely variable in space and time, a robust, space–time observable of ocean noise is difficult to identify. In this work, we derive and verify with data a space–time correlation function of ocean noise between two hydrophones is experimentally demonstrated. Though the sources of ocean noise are uncorrelated, the time-averaged noise correlation function exhibits deterministic waveguide arrival structure embedded in the time-domain Green’s function. A theoretical approach is derived for both volume and surface noise sources. Shipping noise is also investigated and simulated results are presented in deep or shallow water configurations. The data of opportunity used to demonstrate the extraction of wave fronts from ocean noise were taken from the synchronized vertical receive arrays used in the frame of the North Pacific Laboratory (NPAL) during time intervals when no source was transmitting. © 2004 Acoustical Society of America. [DOI: 10.1121/1.1797754]

PACS numbers: 43.30.Pc, 43.50.Rq, 43.60.Fg [WLS] Pages: 1995–2003

Extracting coherent wave fronts from acoustic ambient noise in the ocean

Philippe Roux,\textsuperscript{a)} W. A. Kuperman, and the NPAL Group\textsuperscript{b)}

Marine Physical Laboratory of the Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California 92093-0238

(Received 16 May 2003; revised 24 June 2004; accepted 16 July 2004)

A method to obtain coherent acoustic wave fronts by measuring the space–time correlation function of ocean noise between two hydrophones is experimentally demonstrated. Though the sources of ocean noise are uncorrelated, the time-averaged noise correlation function exhibits deterministic waveguide arrival structure embedded in the time-domain Green’s function. A theoretical approach is derived for both volume and surface noise sources. Shipping noise is also investigated and simulated results are presented in deep or shallow water configurations. The data of opportunity used to demonstrate the extraction of wave fronts from ocean noise were taken from the synchronized vertical receive arrays used in the frame of the North Pacific Laboratory (NPAL) during time intervals when no source was transmitting. © 2004 Acoustical Society of America. [DOI: 10.1121/1.1797754]

PACS numbers: 43.30.Pc, 43.50.Rq, 43.60.Fg [WLS] Pages: 1995–2003

I. INTRODUCTION

Acousticians have used incoherent processing of ambient noise to achieve small scale imaging in the ocean\textsuperscript{1} in the same way as optical imaging is performed from incoherent light. Here, we show that it is feasible to transform a collection of apparently incoherent noise sources into a coherent, large scale, imaging field. Actually, every individual source of noise (e.g., a collapsing bubble) in the ocean generates an acoustic field that is potentially coherent when received between two points after long range propagation. However, this small coherent component at each receiver point is buried in the spatially and temporally incoherent field produced by all the widespread noise sources distributed over the ocean. We demonstrate in this paper that a long-time cross-correlation process extracts coherent wave fronts from ambient noise without the support of any identifiable source. This means that noise could be used as a potential coherent source in the ocean leading to the concept of a self-imaging process. These general results reveal the potential information content of random noise fields in a natural environment.

The sources of ocean surface noise\textsuperscript{2–4} (natural and man-made) as well as the subsequent average spatial distribution of ocean noise\textsuperscript{5–7} have been studied extensively. However, because the instantaneous distribution of all the mutually incoherent sources is extremely variable in space and time, a robust, space–time observable of ocean noise is difficult to identify. In this work, we derive and verify with data a space–time coherence property of surface noise

\textsuperscript{1}Electronic mail: philippe@mpl.ucsd.edu

\textsuperscript{2}J. A. Colosi, B. D. Cornuelle, B. D. Dushaw, M. A. Dzieciuch, B. M. Howe, J. A. Mercer, W. Munk, R. C. Spindel, and P. F. Worcester.
frequency regime of the available data. Without the presence of identifiable events, it is shown that only noise sources aligned along the line between the receivers contribute over a long-time correlation. This paper is structured as follows. In Sec. II, we describe the basic principles of the ambient noise cross-correlation technique in the ocean. Section III presents experimental results using a known shipping source verifying that the residual of the time-averaged correlation function indeed comes from noise sources located in the endfire direction of the receivers. Section IV starts with a theoretical derivation of the ambient noise time-domain correlation function with noise sources distributed either in the volume or at the surface of the waveguide. The case of shipping noise is also explored. Simulations in shallow water point out the similarities and differences with the TDGF. Finally, we present in Sec. V coherent wave fronts obtained from ambient noise data simultaneously recorded on four coplanar hydrophone arrays.

**FIG. 1.** (a) Two arrays are depicted at a separation distance $R$. A schematic of the directivity pattern of the time-domain correlation process between two receivers on each array is projected on the ocean surface. Only a discrete set of lobes have been displayed that correspond to noise sources whose emission angle is equal to $-60^\circ$, $-30^\circ$, $0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. Each angular lobe depends on the central frequency and bandwidth and corresponds to a delay time in the correlation function. For the case of equally distributed ambient noise sources, the broad end-fire directions will contribute coherently over time to the arrival times associated with the TDGF while the contribution of the narrow off-axis sidelobes will average down. For the case of shipping noise, coherent wave fronts emerge only when there is sufficient intersection of the shipping paths with the end-fire beams. However, if there is a particular loud shipping event, it will dominate so that either impractically long correlation times are needed, or discrete events should be filtered out. (b) and (c) The correlation process is done using time-domain ambient noise simultaneously recorded on two receivers in arrays 1 and 2. (d) Spatial temporal representation of the wave fronts obtained from the correlation process between a receiver in array 1 at depth 500 m and all receivers in array 2 separated by a distance $R=2200$ m. The arrival structure of the correlation function is composed of the direct path, surface reflected, bottom reflected, etc., as expected in the TDGF. Because data were taken in 20 min segments, the result is a combination of three segments in order to accumulate the wave-front structure from what turns out to be shipping noise (see Fig. 7 for further explanation). The correlation function is plotted in a dB scale and normalized by its maximum. (e) The same correlation processing is performed on data that have not been recorded at the same time on the two arrays. In (d) and (e), the $x$ and $y$ axes correspond to the time axis of the cross-correlation function and receiver depth, respectively. The correlation functions are plotted in a dB scale and normalized by the maximum of (d).

**II. BASIC PRINCIPLES AND EXPERIMENTAL DEMONSTRATION**

The overall concept is summarized in Fig. 1 in which ambient noise at two receivers [Figs. 1(b) and 1(c)] in array 1 and array 2 are pairwise cross-correlated. In Figs. 1(d) and 1(e), this correlation is a function of delay time and vertical position (depths of receivers in array 2) as is the TDGF between a position in array 1 and the receivers of the array 2. The directivity pattern of the correlation process for a set of incoming angles is schematically projected on the ocean surface [Fig. 1(a)]. For each incident angle, the directivity beam depends on the central frequency and bandwidth (see Sec. III for details) and corresponds to a delay time in the correlation function. This directivity pattern shows how noise sources all over the ocean surface participate to the noise cross-correlation function. Noise sources inside the same directivity beam add coherently while noise sources inside different directivity beams average incoherently (i.e., there is a different delay time associated with each beam). In Fig. 1(a), we see that two broader beams (so called end-fire beams) are
aligned on the axis between the two arrays. Because of their size, these beams yield a larger contribution to the noise correlation function. For example, the two dashed lines in Fig. 1(a) show the coherent contribution from surface noise sources that travel through both receivers. We will show in Sec. III that noise sources inside the end-fire beams provide the residual time-averaged coherence between the two arrays.

In Fig. 1(d) we show a display that is an actual multi-day composite of three 20 min segments of simultaneous recordings of data discussed in Sec. IV. The wave fronts in the display obtained from the cross-correlation process are symmetric in time with respect to the zero time of the correlation function because noise sources were distributed on both sides of the arrays. Note that there is no correlation between data not recorded at the same time [Fig. 1(e)] confirming the hypothesis that coherent wave fronts built up over time from individual noise sources whose acoustic field propagates through both receivers.

The basic difference between the Weaver cavity configuration and ocean noise is the 3D reverberation physics of the cavity versus the two-dimensional ocean waveguide physics of traveling waves in the horizontal direction. For the latter, this means that a ray aligned along the receiver axis will pass through both receiver points by reflection or refraction; however, if the ray has a horizontal component not along the horizontal line between the receivers, it cannot be reflected back to the second receiver and therefore cannot contribute to building up the coherent wave fronts between the receivers. This geometrical interpretation does not apply to the cavity but is still valid in the case of Campillo’s multiple scattering earth model. Indeed, even if the seismic events are not aligned with the two seismometers, we believe that the main contribution to the average correlation function comes from the scatterers present inside the end-fire beams. Indeed, the direct path is built from those scatterers in the upper crust of the earth that behave as secondary sources and re-direct the incident field so that part of the wave travels on a straight line through both receivers.

III. EXPERIMENTAL DEMONSTRATION OF LONG-TIME TWO-POINT CORRELATION PROCESSOR

The underlying physics of this technique relies on long-time cross correlation of ambient noise data. Cross correlation of acoustic data between hydrophones is a very common signal processing tool to detect and locate sources in the ocean. The difference here is that we are not interested in the actual noise sources but in the residual coherence between the hydrophones. This coherence, which corresponds to the temporal arrival structure of the TDGF, is extracted when the cross correlation is performed on long-duration time series. In the case of nonstationary noise sources (surface noise or noise generated by a ship in motion), the signature of the noise sources in the cross-correlated signal averages out and disappears while the coherent paths between the two hydrophones remain.

A separate experimental demonstration of this process has been performed from data simultaneously recorded on two sono-buoys at a few hundred meters from each other in a shallow water environment. Noise was generated during 16 min in the 100–300 Hz frequency interval by a ship whose track is represented in Fig. 2. The two 16-min-long time series are then cross correlated using different time windows (Fig. 3). When the correlation is performed on 1 s duration time series [Fig. 3(a)], the ship track is clearly observed. If the length of the cross-correlated time series is increased to 5, 10, 20, and up to 30 s [Figs. 3(b)–3(e), respectively], the signature of the ship track tends to disappear and the only signal left is obtained when the ship crosses the end-fire main lobes (Fig. 2). This signal exhibits different bottom and surface-reflected paths classically found in a shallow water environment. We will show in Sec. IV that it converges to the arrival structure of the TDGF between the two sono-buoys when averaged over several different ship tracks. Obviously, we observe in Figs. 3(a)–3(e) that the longer the correlation window, the higher the signal-to-noise ratio because more acoustic sources participate coherently to correlation function. Then, for longer time series [Fig. 3(f)], the correlation pattern does not change because no more coherent sources were present in the signal. Assuming that the speed of the ship was constant during the track, it generates a uniform density of sources over time. For long time windows, the signal-to-noise ratio of the correlation process can be defined as the ratio of the number of coherent versus incoherent sources inside the recording time window. Following the geometrical interpretation developed in Sec. II, this ratio corresponds to the area enclosed by the end-fire beam to a non-end-fire beam.

The directivity pattern \( B(\theta, \theta_0) \) in the direction \( \theta \) of the correlation function between two receivers for an incident wave in the direction \( \theta_0 \) can be written as

\[
B(\theta, \theta_0) = \int \frac{\omega - \Delta \omega/2}{\omega + \Delta \omega/2} \left[ 1 + \exp \left( \frac{\omega' R}{c} \cos(\theta) \right) \right] \\
\times \left[ 1 + \exp \left( -i \frac{\omega' R}{c} \cos(\theta_0) \right) \right] d\omega',
\]

where \( R \) is the distance between the two receivers, \( \omega = 2 \pi f \).
the central angular frequency, and $\Delta \omega$ the frequency bandwidth. The angle $\theta$ and $\theta_0$ are defined with respect to the axis of the two receivers. After development and integration, we obtain the following from Eq. (1).

1. If $\theta_0 \neq 0$ and $\delta \theta \ll \theta_0$,

$$B(\theta_0 + \delta \theta, \theta_0) \approx 1 - \frac{\delta \theta^2}{2} \left( \frac{2 R \sin(\theta_0)}{c} \right)^2 \left( \omega^2 + \frac{\Delta \omega^2}{12} \right).$$

(2)

2. If $\theta_0 = 0$ or $\theta_0 = \pi$ and $\delta \theta \ll \theta_0$,

$$B(\delta \theta, 0) = B(\delta \theta + \pi, \pi) = 1 - \frac{\delta \theta^4}{8} \left( \frac{R}{c} \right)^2 \left( \omega^2 + \frac{\Delta \omega^2}{12} \right).$$

(3)

Equations (2) and (3) show that the end-fire beams (for $\theta_0 = 0$ or $\theta_0 = \pi$) are broader. It follows that the ratio of the area enclosed by the end-fire beam to a non-end-fire beam is

$$\text{Ratio}(\theta_0) = \sqrt{\frac{\omega}{c} R \sin(\theta_0) \left( 1 + \frac{1}{12} \left( \frac{\Delta \omega}{\omega} \right)^2 \right)^{1/4}}.$$  

(4)

Taking $R \approx 650 \, \text{m}$, $f = \Delta f = 200 \, \text{Hz}$, $c = 1500 \, \text{m/s}$ we get a maximum signal-to-noise ratio of 25 dB, which is in good agreement with the data [Fig. 3(e)]. The maximum signal-to-noise ratio is obtained for a time window $T \sim 30 \, \text{s}$ that is approximately the time spent by the ship in the end-fire beam.

IV. THEORY AND SIMULATION

The analogy to the Weaver volume cavity noise would ideally be ocean volume noise sources uniformly distributed throughout the water column. However, ocean noise is dominated by surface noise sources that are typically uniformly distributed over the ocean surface as one goes to higher frequencies ($\sim 1 \, \text{kHz}$, see Ref. 2). For the lower frequency case ($\sim 100 \, \text{Hz}$), data are dominated by shipping noise, and while the concept remains the same, the relative amplitudes of the wave fronts that become observable will be dependent on the specific shipping distribution during the recording time interval. As a matter of fact, it is shown here by theory and

FIG. 3. Representation of the temporal evolution of the time-domain cross-correlation function between the two sono-buoys along the 16-min-long ship track. The $x$ and $y$ axes correspond to the time axis of the correlation function and the ship position, respectively. The duration of the time windows on which the cross correlation is performed is (a) 1 s, (b) 5 s, (c) 10 s, (d) 20 s, (e) 30 s, and (f) 40 s. Each cross-correlation pattern is normalized by its maximum. The color scales are in dB.

FIG. 4. Schematic of the waveguide in which simulations are performed. A source receiver in $z_1 = 100 \, \text{m}$ and an array of receivers are located at a distance $R = 2500 \, \text{m}$ from each other in a 150-m-deep shallow water environment. The surface noise sources are at depth $z' = 1 \, \text{m}$. The sound-speed profile decreases linearly from 1500 m/s at the surface to 1480 m/s at the bottom. The bottom sound speed, density, and attenuation are 1800 m/s, 1800 kg/m$^3$, and 0.05 dB/ft, respectively.
simulation that shipping produces similar results to the surface noise case but that distant shipping emphasizes more horizontally traveling wave fronts than nearby ships for which the wave fronts are more vertical.

The ocean acoustic environment is typically treated as a waveguide\textsuperscript{16} such that the propagation between points 1 at depth \(z_1\) and 2 at depth \(z_2\) separated by horizontal range \(R\) (Fig. 4) is given, in its simplest form, by a normal mode expansion:

\[
G_w(R,z_1,z_2) = \frac{iS(\omega)}{4\rho} \sum_n U_n(z_1)U_n(z_2)H_0^{(1)}(k_nR), \tag{5}
\]

where \(U_n(z)\) is the depth-dependent eigenfunction associated with wave number \(k_n\), \(\rho\) is the density at the source location, and \(S(\omega)\) is the source spectrum. When integrated over the frequency bandwidth, Eq. (5) becomes the TDGF:

\[
G_s(R,z_1,z_2) = \int d\omega G_w(R,z_1,z_2)\exp(-i\omega t). \tag{6}
\]

The wave-front structure of the Green’s function results from modes with similar group speeds constructively interfering over frequency; mathematically the wave fronts can be shown to emerge, for example, from a stationary phase evaluation\textsuperscript{17} of Eq. (6) that results from the condition \(d(k_nR - \omega t) = 0\).

### A. Volume-noise case

In order to demonstrate the connectivity between the volume-cavity result of Weaver and the correlation of ocean waveguide noise as formulated by Kuperman and Ingenito,\textsuperscript{6,16} we first modify the latter theory to include volume sources. This shows how the waveguide TDGF emerges in analogy to the cavity case. Following Ref. 6, the modal decomposition in the frequency domain of the volume noise cross-correlation function between points 1 and 2 is

\[
C_w(R,z_1,z_2) = \frac{i\pi Q^2(\omega)}{k^2(\omega)} \int \frac{1}{\rho^2(z')} dz' \sum_{nm} U_m(z')U_n(z_1) \\
\times U_m(z_2)U_n(z_2) \frac{1}{k^2_n - k^2_m} \\
\times [H_0^{(1)}(k_nR) - H_0^{(1)}(-k_nR)]. \tag{7}
\]

where \(Q^2(\omega)\) is the power spectrum of the noise sources and \(z'\) the depth of the noise sources. To obtain Eq. (7), we assumed that each sheet of noise sources at depth \(z'\) are uncorrelated and that noise sources are uniformly distributed in the whole water column. The spatial integration over the depth of the waveguide can be performed using the orthogonality condition of modes

\[
\int \frac{U_n(z')U_m(z')}{\rho(z')} dz' = \delta_{nm}.
\]

Doing so, we neglect the tilt of the mode in the bottom of the waveguide and we assume a constant density in the water column \(\rho(z') = \rho\). We also suppose that \(k_n\) is a complex number of the form \(k_n = K_n + i\alpha_n\) (with \(K_n \gg \alpha_n > 0\)), where \(\alpha_n\) is the modal attenuation coefficient. The final expression for the ambient noise correlation function is then:

\[
C_w(R,z_1,z_2) = \frac{\pi Q^2(\omega)}{4\rho k^2(\omega)} \sum_n U_n(z_1)U_n(z_2) \frac{1}{\alpha_n K_n} \\
\times [H_0^{(1)}(k_nR) - H_0^{(1)}(-k_nR)]. \tag{8}
\]

After frequency integration, the time-domain ambient noise correlation function is obtained from \(C_s(R,z_1,z_2) = \int d\omega C_w(R,z_1,z_2)\exp(-i\omega t)\). The two Hankel functions in Eq. (8) represent two wave fronts traveling between receivers 1 and 2 in opposite directions. Physically speaking, the two wave fronts arise from a uniform volume noise distribution so that at any point, noise is coming from all directions. Invoking the modal normalization condition, \(\int [U^2_n(z)/\rho(z)] dz = 1\), the amplitude factor

\[
\alpha_n K_n = \int \alpha(z) \frac{\omega}{c(z)} \frac{U^2_n(z)}{\rho(z)} dz
\]

(where \(\alpha(z)\) is the depth-dependent absorption\textsuperscript{16}) is slowly dependent on mode number. Thus, the modal decomposition in Eq. (8) is very close to the Green’s function decomposition as written in Eq. (5). This means that the correlation function obtained from volume ambient noise recorded at two receivers in a waveguide is a good approximation of the Green’s function between the two points. The reasoning done through Eqs. (7) and (8) is the waveguide equivalent of Weaver’s cavity approach in which he supposed the modes equipartition due to an uniform noise distribution in the cavity.

### B. Surface-noise case

Assuming a sheet of noise sources located at a given depth \(z'\) only, Kuperman and Ingenito obtained the following expression for the noise correlation function:

\[
C_w(R,z_1,z_2) = \frac{\pi Q^2(\omega)}{4\rho k^2(\omega)} \sum_n U_n(z_1)U_n(z_2) \\
\times [U^2_n(z')/(H_0^{(1)}(k_nR) - H_0^{(1)}(-k_nR))]. \tag{9}
\]

In the case of noise sources distributed at the surface of the ocean, a monopole source below the pressure release ocean surface behaves as a dipole structure. The amplitude factor \(U^2_n(z')/\alpha_n K_n\) in Eq. (9) results then from a combination of the dipole behavior of the noise sources and the effect of attenuation over long ranges. Formally, the factor arises because there is no integral over the depth of sources as in the volume case; that is the only difference between the volume and surface cases. Since this amplitude term will not affect the stationary phase argument that synthesizes the wave fronts,\textsuperscript{18} the time-domain surface noise correlation function \(C_s(R,z_1,z_2)\) will exhibit the same wave-front structure as the two point Green’s functions though the amplitude of the wave fronts will be different. We therefore conclude that
after the temporal averaging underlying Eq. (9), we obtain coherent wave fronts from ocean surface noise. However, these coherent wave fronts only constitute an approximation of the TDGF; the excitation of each mode being weighted by a dipole shading. This approach is similar to Campillo’s result in the sense that only the Rayleigh wave was reconstructed between two seismometers using the late coda of seismic events. The longitudinal and shear waves that classically participate to the TDGF between two points at the earth surface are missing in the correlation function because they are not properly excited by the scatterers present in the upper crust of the earth.

In Fig. 5, we confirm this theoretical approach with simulations using the spectral model performed in a shallow water environment in the 50–150 Hz frequency bandwidth (Fig. 4). The time-domain correlation function of surface noise [Fig. 5(a)] is compared to the actual Green’s function [Fig. 5(b)] between one source-receiver and an array of receivers. As expected, the same coherent wave fronts are observed but the amplitude of the higher-order reflected paths is different. Consistent with the above-mentioned results, the Green’s function computed with a vertical dipole source instead of an omnidirectional monopole excitation [Fig. 5(c)] shows an obvious similarity with the time-domain correlation function.

C. Shipping-noise case

For the pure shipping case, the correlation function results from the product of two Green’s functions each of the form given in Eq. (5) but evaluated at the radial distances \( R_1 \) and \( R_2 \) between the ship and receivers 1 and 2, respectively:

\[
C_{\text{w}}(R; z_1, z_2) = \frac{\widetilde{Q}^2(\omega)}{\rho^2} \sum_{n,m} U_n(z') U_m(z') \frac{U_n(z_1) U_m(z_2)}{\sqrt{k_n k_m}}
\]

\[
\times U_m(z_2) H_0^{(1)}(k_n R_1) H_0^{(1)}(-k_m R_2),
\]

where \( \widetilde{Q}^2(\omega) \) is the power spectrum of the shipping noise. As explained in Sec. III, the correlation process emphasizes the contribution from ships located in the end-fire beams.

When a ship is located in the end-fire direction of the two receivers, we have \( |R_1 - R_2| = R \). Evaluating the average contribution from a ship involves integrating Eq. (10) over the length \( L \) of the ship track inside the end-fire beam [see Fig. 1(a)] yielding an amplitude term sin \((k_n - k_m)(L/2)\) in the form of a term \( \sin([k_n - k_m] x) \)

\[
\langle C_{\text{w}}(R; z_1, z_2) \rangle \propto \frac{\widetilde{Q}^2(\omega)}{\rho^2} \sum_n \frac{U^2(z')}{\sqrt{k_n}} U_n(z_1)
\]

\[
\times U_m(z_2) H_0^{(1)}(k_n R_1) H_0^{(1)}(-k_m R_2),
\]

where the angular brackets \( \langle \rangle \) refer to the average over an accumulation of ship events. When ship paths cover the whole ocean surface, the propagating wave in Eq. (11) is changed into two propagating waves in opposite directions as in Eq. (9). Equations (11) and (9) then become very similar in the sense that they both correspond to an amplitude-shaded approximate version of the TDGF.

Though we get the same forms for either broadband shipping or surface ambient noise, the signal-to-noise ratio (emergence of the wave fronts) of the two cases will be different as the schematic of Fig. 1 indicates and as discussed further in the following. For example, as opposed to ambient noise sources, shipping events could be possibly identified, normalized in amplitude, and integrated separately in the correlation function. This would balance the contribution of close/faraway ships with loud/low source levels. Furthermore, since shipping is episodic, it would be helpful to elimi-
nate high amplitude events from the data and thereby “ho-
mogenize” the noise in order to enhance the convergence to
the expected correlation function. The best data set displays
no specific events and is approximated well by the theory of
uniformly distributed surface noise sources.

Simulations of the above-noted processes are given in
Figs. 6 and 7 in which the TDGF is numerically computed
with the surface noise correlation function or the shipping
events correlation function. The ocean environment is close
to the experimental configuration described in Sec. V

FIG. 6. Simulations of the Green’s function (TDGF) and the time domain
correlation function in an oceanic environment close to the experimental
data. (a) Spatial-temporal TDGF from a source at depth 800 m to an array at
da distance of 2400 m. (b) The same TDGF but source shaded by a dipole
pattern similar to the radiation pattern of near-surface sources. (c) Surface
noise correlation function between the same position of the source and the
array showing a close similarity to (b). The x and y axes correspond to the
time axis of the correlation function and the receiver depth, respectively.
The color scales are in dB.

Simulations of the above-noted processes are given in
Figs. 6 and 7 in which the TDGF is numerically computed
with the surface noise correlation function or the shipping
events correlation function. The ocean environment is close
to the experimental configuration described in Sec. V
[Fig. 8(a)]. As in Fig. 5, when the Green’s function [Fig. 6(a)] is
combined with the directivity of a dipole [Fig. 6(b)], the
depth-time dependent pattern of the Green’s function and the
noise correlation function [Fig. 6(c)] look similar. However,
the amplitudes of the wave fronts are still different because a
dipole at 600 m does not have the same directivity pattern as
a dipole at the surface in a depth-dependent sound speed
profile.

Further, Fig. 7 refers to an end-fire ship track (source
contributions from other directions vanish over long correla-
tion time window, as shown in Fig. 3) at different ranges. In
Figs. 7(a)–7(c), we observe that the wave-front arrivals have
different amplitude emphasis—shorter ranges favoring more
vertical paths in the correlation function. Figure 7(d), which
is a composite of the various ranges, exhibits, as expected,
the same arrival structure as in Fig. 6(a). Hence, in the ship-
ing case, a collection of random shipping events will fill in
the whole TDGF pattern over time. Roux and Fink20 have
theoretically examined the case of averaging correlations of
deterministic sources over depth that theoretically results in
modal equipartition.12,14 However, as an impractical matter
in ocean acoustics, it requires the insertion of active sources
and is more akin to the above-discussed hypothetical volume
source problem.
V. EXPERIMENTAL RESULTS WITH HYDROPHONE ARRAYS

The actual measurement and signal processing that correspond to the theoretical results of Eq. (9) is done in the time domain where the correlation function $C_{12}(t)$ is measured using $C_{12}(t) = \int S_1(\tau)S_2(t+\tau)\,d\tau$, where $S_1(t)$ and $S_2(t)$ are the ambient noise received on receivers 1 and 2 at time $t$. Note that the correlation processing requires data measurement that have a common clock time. We use data of opportunity from the NPAL program\(^\text{21}\) originally taken for other purposes. Data correspond to different sets of 20 min simultaneous recording of ambient noise on four vertical arrays, filtered between 70 and 130 Hz. Despite the obvious presence of shipping noise in this frequency bandwidth, high amplitude signals were not identifiable in the spectrograms at the receivers. Using four coplanar arrays [Fig. 8(a)] enables us to measure the noise correlation function with respect to the travel time separation between one receiver in array 1 and all receivers in arrays 2–4 as shown in Figs. 8(b)–8(d). Note in Figs. 8(a) and 8(d) that array 4 has twice as many elements as arrays 1–3. We observe from the correlation lag times that we have extracted wave fronts as they would propagate from a point source to ranges of 1700, 2400, and 3500 m, respectively. We also show that we recover similar wave fronts for the opposite direction by correlating one receiver in array 4 with all receivers in the other arrays. Here we see traveling wave fronts in the direction of the arrow, opposite case (b), (c) and (d), as if they emanated from the receiver in array 4. The wave fronts for this direction are more vertical because of the slope effect—further confirming the correct extraction of the arrival structure of the TDGF. In (b)–(g), the $x$ and $y$ axes correspond to the time axis of the correlation function and the receiver depth, respectively. The color scales are in dB.

FIG. 8. The noise-derived amplitude-shaded TDGF extracted from the NPAL data. (a) The array geometry indicating a sloping bottom. Note that array 4 is made of twice as many elements as arrays 1–3. (b)–(d) Time-domain correlation functions between a receiver at depth 500 m in array 1 and all receivers in the other arrays. Traveling wave fronts are clearly observed in the direction of the arrow as if they emanated from the receiver in array 1. (e), (f), and (g) Time-domain correlation functions between a receiver at depth 500 m in array 4 and all receivers in the other arrays. Here we see traveling wave fronts in the direction of the arrow, opposite case (b), (c) and (d), as if they emanated from the receiver in array 4. The wave fronts for this direction are more vertical because of the slope effect—further confirming the correct extraction of the arrival structure of the TDGF. In (b)–(g), the $x$ and $y$ axes correspond to the time axis of the correlation function and the receiver depth, respectively. The color scales are in dB.

The sloping environment results in the asymmetry between the two directions, i.e., upslope increases the reflection angle.

From the physical picture and the measurements presented above we can derive insight into the major components governing the rate of emergence of the coherent wave fronts for a homogeneous distribution of random sources. However, we can only draw some qualitative insight into our specific data of opportunity that is dominated by episodic shipping. Consider two receivers separated by a distance $R$. The signal-to-noise ratio (SNR) at the output of the correlation depends on three physical phenomena. First, it depends on the part of the signal that contributes to the correlation compared to the uncorrelated ambient noise, either acoustic or electronic. The uncorrelated acoustic ambient noise corresponds to the field that does not reach the two receivers because of the medium attenuation or above critical grazing angle transmission into the bottom. That is, the noise field consists of a local and some nonlocal components which vary with overall attenuation. Second, the correlation function is built from the contribution of noise sources located in the end-fire beams versus noise sources in the non-end-fire beams [cf. Eq. (4)]. Last, for a bandwidth $\Delta f$, the SNR associated with a correlation process grows with recording time $T$ and is given by $\sqrt{T\Delta f}$. In any event, the total SNR at the end of the correlation process is related to the
time bandwidth product, the spatial structure of the correlator as described in Eq. (4), and an environmental factor (expressions for local versus long range contributions given in Refs. 23 and 24), the latter typically being not known for an arbitrary location. Without specific environmental knowledge, the rate of emergence TDGF is a measured parameter that can be used to estimate the bottom geophysics (since volume attenuation is typically known). These arguments are for homogeneous ambient noise; the additional complication of shipping noise requires a long enough measurement period that demonstrates convergence.

VI. CONCLUSION

Our results demonstrate the potential information content of a random noise field. In particular, we have demonstrated through theory and data analysis that we can recover coherent deterministic wave fronts related to the structure of the time domain Green’s function using measurements of ocean ambient noise between vertical arrays. In the NPAL data analysis, we have used the 20 min data blocks that were available and it is not likely that we have done sufficient time averaging for the optimal result for a shipping dominated environment. Since shipping noise is dominant at lower frequencies (<1000 Hz), we expect that high frequency will yield the most complete, uniformly converging, two-sided wave fronts predicted by theory. Further experiments in this higher frequency range are expected to provide insights into the convergence time of this process. The results presented here are a first step toward passive tomographic imaging.

ACKNOWLEDGMENTS

This research was supported by the Office of Naval Research. The authors would like to thank Erin Oleson for her help in acquiring sonobuoy data during a Scripps experiment conducted by John Hildebrand’s group and W. S. Hodgkiss, H. C. Song, and K. Sabra for valuable discussions.

3 R. K. Andrew, B. M. Howe, J. M. Mercer, and M. Dzieciuch, “Ocean ambient sound: Comparing the 1960s with the 1990s for a receiver off the California coast,” Acoust. Res. Letters Online, 3, 65—70 (2002)]. In this paper, Wenz’s results have been compared against more recent data.
5 D. Ross, Mechanics of Underwater Noise (Peninsula, Los Altos, 1987). Estimates of shipping densities given extrapolate to a distribution with more than 1 ship per square degree which suggests (and has been confirmed for many years in the field of Underwater Acoustics) that except for nearby specific ships tracks, distant shipping can be considered to be smeared out over the large surface of the ocean, albeit with a directional dependence.
19 The North Pacific Acoustic Laboratory (NPAL) experiments were designed to study coherence of acoustic signal propagating long distances in the ocean. The acoustic source was 3000 km from the arrays. The NPAL group provided us with noise data from their receiver array during times when their source was not transmitting. Their array technology is the same used in the Acoustic Thermometry of the Ocean Climate experiments [ATOC Consortium, “Ocean climate change: comparison of acoustic tomography, satellite altimetry and modeling,” Science 281, 1327—1332 (1998)].
20 The data occasionally showed distinct correlation building up over 20 s intervals. A typical ship might have a source level of 165 dB and at 100 Hz, a transmission loss of order 80 dB (see Ref. 5, p. 12). The ambient noise is about 90 dB so that the SNR ≈ —5 dB. With a time-bandwidth product $\omega_0(Hz)\times \tau (s) \sim 30$ dB, we definitely should see occasional distinct correlation patterns compressed by the ship bearing projection onto the line between the two receivers.