INTRODUCTION

Various forms of acoustic imaging are playing an increasing role in research and everyday life. It is becoming common practice for a pregnant woman to have a "sonogram" of her baby to be sure that all is well. Statistical quality control requires rigorous testing of manufactured items; much of the testing is done using acoustic nondestructive evaluation (NDE). In science and research, acoustic microscopes are being used that can often give a sharper image than optical microscopes. In clinical applications, research is providing simultaneous quantitative images of the sound velocity, attenuation, and fluid velocities. In the field of fluid dynamics, acoustic techniques are being used to map the temperature and velocity fields in laboratory to ocean scale flows.

The invention of x-ray tomography by Hounsfield in 1972 stimulated the current interest in all kinds of tomography. Since then the instrumentation and computer hardware and software have been improved so that accuracies of one part in 1000 are achievable from \(10^6\) data points. In the field of medicine, tomography has let doctors study the interior of a patient's body with precision and safety. While x rays are used to produce images of the x-ray attenuation coefficient, other forms or sources of energy including radio waves, magnetic resonance, positrons, radioisotopes, and sound are used to image different parameters.

Acoustic imaging can be loosely divided into tomographic and nontomographic systems. Nontomographic imaging generally consists of a single view of an object and presents the raw data with little or no processing. The resulting qualitative images highlight regions of strong scattering associated with large gradients in the refractive index or, equivalently, regions of high reflectivity, such as distinct boundaries. However, the "correct" interpretation of these qualitative images is largely dependent on the skill of a trained analyst. Nontomographic imaging is widely used in medicine and NDE.

In contrast, tomographic imaging is a distinctly two-step process. In its most pristine form, the object is first probed from many different angles. The measured data from the various views are then processed and combined to yield a quantitative reconstruction of some physical parameter such as the acoustic index of refraction. Tomographic reconstruction is therefore a much more experimentally and computationally intensive procedure, but the quantitative images that are generated...
can reveal subtleties that are not recovered by nontomographic methods.

In practice, the classification of a particular acoustic imaging system is not always clear. Many practical systems combine both tomographic and nontomographic features. For example, inherently nonquantitative medical images taken from different angles may be superimposed to create a superior image. In medical or geophysical applications of tomography, the number of views available may be far fewer than the number that are required for ideal inversion. The type of system used for a particular application depends on a variety of factors. In nondestructive evaluation, for example, the mere detection of a flaw may be sufficient, and quantitative information may not be needed.

The tomographic reconstruction process is illustrated in Fig. 1. In the simulation, a system of four cylinders is probed by an acoustic plane wave. An image based on a single view is inadequate for recovering the system. However, as the images generated from additional views taken at other angles are superimposed, the reconstruction is improved and the individual cylinders become apparent.

Acoustic tomography is related to several other topics covered in this volume. Acoustic microscopy (see MICROSCOPY, ACOUSTICAL) can be used in detecting surface cracks in NDE. Some forms of microscopy are in fact tomographic, but are beyond the scope of this article. The coherent field measurements required for diffraction tomography can be interpreted as acoustic holograms. Indeed, generalized acoustic holography (q.v.) can form the basis of a variety of both tomographic and nontomographic inverse scattering algorithms (Porter, 1989). Finally, because of the large amount of recorded data needed for tomographic inversion, efficient acoustic signal processing (see SIGNAL PROCESSING) techniques are required.

The contents of this article are divided primarily by imaging technique rather than by application. By taking this approach, we stress the similarity of the imaging problem for such seemingly disparate fields as medicine, geophysics, nondestructive evaluation, and oceanography. The first section briefly reviews nontomographic techniques as background for the tomographic methods discussed in the balance of the paper. The next section describes conventional straight-ray tomography. In this method, the probing energy is assumed to travel along straight-ray paths through the object. Conventional tomography also illustrates the importance of multiple views for quantitative inversion. The third section discusses diffraction tomography. In diffraction tomography, the probing wave is assumed to obey a wave equation. Recent encouraging experimental results in dif-
fraction tomography are discussed. Examples of current research from medicine, geophysics, and small-scale oceanography are included.

Straight-ray and diffraction tomography are similar because in many applications reconstructions are often "agnostic"; i.e., there is sufficient data that nothing need be assumed about the field being reconstructed. Section 4 deals with ocean acoustic tomography, in which the rays are refracted by the ocean sound channel. Because of a paucity of data, a priori information is necessary to constrain the reconstruction. Concluding remarks are in Sec. 5.

1. NON TOMOGRAPHIC IMAGING

Most methods of acoustical imaging currently used in clinical and NDE applications are non-tomographic and are based on reflection or pulse-echo techniques. The different methods have historically been assigned letter designations: A-scan, B-scan, C-scan, etc. [Figs. 2(a) and 2(b)]. Many of the commercial systems incorporate some aspects of each method (Schuler et al., 1984).

The A-scan is based on "time of flight": A pulse is transmitted and bounces off a scattering or reflecting boundary and the echo is detected, often by the transmitting element which also acts as a receiver [Fig. 2(a)]. The time between transmission and reception of scattered energy gives the range to the reflecting surface (assuming a constant speed of sound). The amplitude of the echo provides information on the strength of the scattering. The time corresponding to a large-amplitude peak is used to locate a major scattering boundary. It could be the interface between different organs in the body, a wall of a pipe, or the ocean bottom. Weaker amplitudes would be representative of, for instance, the relative severity of a flaw in a weld or casting, or scatter from schools of fish or zooplankton, or sediment layers beneath the ocean floor.

Time modulated (T/M) mode interrogation involves a succession of A-scan measurements taken at a fixed transducer location in even time increments. The output gives the
It is difficult to couple acoustic waves from air into a solid because most of the energy is reflected. Furthermore, the faults present in an object may vary widely in size, shape, and location; this diversity can complicate the interpretation of the output.

A similar procedure is also found in geophysics (see SEISMOLOGY). The objective in seismic inversion is to determine the structure of the earth beneath its surface. This information is crucial in oil exploration or in the disposal of nuclear waste. Low frequencies of less than 100 Hz are used to obtain information at depths of several kilometers. As with NDE, shear and surface waves are generated. Seismic imaging is simplified by the a priori assumption that structures vary primarily in depth by geologic layer.

The C-scan can operate in either the reflection or transmission mode. As a transmitter is moved over a plane, a receiver is also moved and the received intensity as a function of position is recorded. This is equivalent to a simple x-ray picture since both provide a "shadowgraph" of the object.

Doppler ultrasound uses the change in apparent frequency between the transmitted and received signals to measure relative motion. The technique is often used in medical applications to monitor blood flow. A typical transmission frequency is 3 MHz and the shift in frequency due to blood flow is usually in the audio range. Consequently, after demodulation the output signal is often sent to an audio speaker allowing the clinician to "hear" the blood flow. Continuous-wave Doppler systems require separate transducers for transmission and reception and give no information about the location of a flow. Pulsed Doppler systems do give range data, but require more elaborate timing and demodulation circuitry.

2. STRAIGHT-RAY TOMOGRAPHY

The nontomographic systems of the previous section operate primarily in the pulse-echo mode. Many tomographic systems operate in the transmission mode; the object is between the transmitter and receiver. In propagating through the object, the probing wave is perturbed in accordance with the properties of the medium. The simplest prop-
agitation model is to assume that the transmitted energy travels along straight ray paths through the object. This model permits the derivation of relatively simple inversion algorithms that parallel those common to x-ray tomodraphy.

For simplicity, we begin with a two-dimensional imaging problem and later generalize to the full three-dimensional case. The typical experimental configuration is shown in Fig. 4. The object function $F(x, z)$ describes some physical parameter of an object such as the attenuation rate or the index of refraction that we wish to determine. For this view, the object is probed by energy traveling along a straight ray in the $z$ direction. It is assumed that the measured quantity (such as intensity or travel time) is related to the geometric projection $P_0(x)$ of the object. The geometric projection at this angle is related to the object function by

$$ P_0(x) = \int_{-\infty}^{\infty} F(x, z) dz. \quad (1) $$

The objective is to recover the unknown function $F(x, z)$ from projections measured at many different angles. If the data are measurements of intensity, then $F(x, z)$ is the attenuation field; if the data are measurements of travel time, $F(x, z)$ is the reciprocal of the sound speed. From a practical point of view, the projection can only be sampled at a finite number of points corresponding to a finite number of rays.

2.1 Projection-Slice Theorem

It is instructive to examine the relationship between the projection and the object function in the Fourier domain. Because of its relative simplicity, the analysis for the geometry shown in Fig. 4 is now presented. Defining the Fourier transform of the projection as $\tilde{P}_0(k_z)$, we find from Eq. (1) that

$$ \tilde{P}_0(k_z) = \int_{-\infty}^{\infty} P_0(x) e^{-i k_z z} dx 
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, z) e^{-i k_z z} dxdz. \quad (2a) $$

Defining the two-dimensional transform of the object as

$$ \tilde{F}(k_x, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, z) \times e^{-i k_x x} e^{-i k_z z} dxdz, \quad (2b) $$

it follows that

$$ \tilde{P}_0(k_z) = \tilde{F}(k_x, k_z=0). \quad (2c) $$

Equation (2c) indicates that by taking the one-dimensional transform of the measured projection, we recover the two-dimensional transform of the object evaluated in Fourier space along the line $k_z = 0$. This result is easily generalized to an insonification angle $\phi$ as shown in Fig. 5. This fundamental relationship is referred to as the Projection-Slice theorem: The one-dimensional transform of the projection taken at angle $\phi$ maps onto a radial line at angle $\phi$ in the two-dimensional Fourier space of the object.

It is apparent from Fig. 5 that if we could probe the object at all possible directions over 180°, we would "fill" Fourier space and could then, in principle, recover the object function $F(x, z)$ by an inverse two-dimensional transform. The figure also illustrates a concept relevant to all types of tomography: A single view provides only limited information about an object and many views are necessary for an accurate quantitative reconstruction.

In practice, the reconstruction is usually performed in the spatial domain. Each projection is first filtered. Alternate filtering schemes are available, but the idea is to compensate for the relatively sparse sampling of $\tilde{F}(k_x, k_z)$ at high spatial frequencies when a finite number of projections are measured (Horn, 1978). These filtered projections are
The Projection-Slice theorem. The one-dimensional projection $P_\phi(\xi)$ maps onto a line in two-dimensional Fourier space of object $F$.

2.2 Example of Straight-Ray Tomography

Schreiman et al. (1984) conducted a study of ultrasound-transmission computed tomography of the breast to evaluate its capability for detecting anomalies. The technique measures both the speed of sound and the attenuation within the breast by measuring the time of flight and reduction in amplitude of sound pulses. There was enough variability between patients in the imaged parameters that relative rather than absolute values were required; reference values were taken from "normal" adjacent tissue.

The scanning apparatus consisted of four focused transmitters and four receivers operating at 5 MHz; the corresponding acoustic wavelength was approximately 0.3 mm. Each transducer was 10 mm in diameter and separated from its neighbor vertically by 14 mm (Fig. 6). Using a translate/rotate scan, each transducer pair obtained a set of data used for reconstructing images in its plane of the breast. At each of 60 viewing angles, the instruments were translated horizontally to construct a projection made up of 200 parallel rays. A second set of scans was made with the transducers lowered 7 mm. This resulted in data for eight planes, 7 mm apart and approximately 3 mm thick. The entire procedure lasted about 5 min. Data were stored on magnetic media and processed off-line.
The arrival time of the sound pulse at the receiver was measured using threshold detection of the signal after it had been passed through a full-wave rectifier and a bandpass filter. The amplitude of the received signal was estimated by integrating the first two microseconds of the signal after detection. Although this latter method averages the attenuation coefficient over frequency, its virtue is simplicity.

An example of the ultrasonic tomographic images is shown in Fig. 7. The reconstruction was done using the filtered backprojection method. It was found that malignant lesions could be identified by various ranges of sound speed and attenuation. The tumor shown has a low sound speed and a high attenuation. More work needs to be done to correlate tomographic results with tissue malignancy. It would also be interesting to see what effects ray bending and diffraction would have on these images. For additional examples of ultrasonic tomography as applied in medicine, the reader is referred to Robinson and Greenleaf (1985).

Tomographic techniques can also be applied to seismic data. Typically several boreholes are drilled between which either acoustic or electromagnetic signals are transmitted. Reconstruction methods often rely on a priori information and reduce to the solution of a system of equations. A similar procedure for the oceanographic case is discussed in detail in Sec. 4. For additional examples from seismology, see Daily (1986).

3. DIFFRACTION TOMOGRAPHY

The conventional tomography algorithms discussed in the previous section were largely developed for x-ray tomography (see X-RAY IMAGING: RADIOLOGY AND TOMOGRAPHY). The central assumption of the method, i.e., that the probing energy travels along straight ray paths, is certainly true for x rays. The successful application of such techniques to acoustics, however, has been limited. In acoustics, diffraction effects often cannot be neglected. Diffraction occurs whenever the acoustic wavelength is comparable to spatial scales associated with changes in the index of refraction of the medium. In diffraction tomography, the measured information is treated like a scattered field governed by the wave equation. We first derive an expression for the scattered field and then show how it can be used to reconstruct an object.

3.1 Scattering Problem

The two-dimensional scattering problem is shown in Fig. 8(a). A scatterer characterized by the index of refraction \( n(x) \) is probed by a known incident acoustic plane wave. For a source-free region, we assume that the total acoustic pressure wave \( \psi \) satisfies the linearized, time-harmonic \( \exp(-i\omega t) \) Helmholtz equation:

\[
[V^2 + k^2 n^2(x)]\psi(x) = 0, 
\]

where \( k = \omega/C_0 \) is the wave number and \( C_0 \) is the speed of propagation in the background medium. In general, the index of refraction can be real or complex, the latter applying when the scatterer is lossy. Outside the object, the index of refraction is equal to that of the homogeneous background and \( n(x) = 1 \).

The total field \( \psi(x) \) outside the object can be written as the sum of the known incident field \( \psi_i(x) \) and a scattered field \( \psi_s(x) \). The incident field is that which would propagate in the absence of the scatterer. The scattered field contains the perturbations in the total
Acoustical Tomography

FIELD 8. Diffraction tomography. (a) An incident plane wave probes the unknown scatterer $F(x, z)$. The measured field is governed by the wave equation. (b) The Fourier diffraction theorem. The one-dimensional transform of the scattered field $\psi_s(x)$ maps onto a semicircle in the two-dimensional Fourier space of the object $F$. The radius of the semicircle is the wave number $k$ of the incident wave.

field due to the object and is given by (Ishimaru, 1978)

$$\psi_s(x) = \int_V F(x') \psi(x') G_j(x, x') dx'$$  \hspace{1cm} (4)

Here $F(x) = k^2[n^2(x) - 1]$ is the scattering function, $V$ is the volume of the object, and $G_j$ is the free-space Green's function. The objective is to recover the scattering function from measurements of the scattered field. This task is complicated by the presence of the unknown total field $\psi(x)$ in the integrand, so simplifying approximations are often made. Equation (4) can be linearized by approximating $\psi$ in the integrand with the known incident field $\psi_i$. This is the so-called (first) Born approximation and is valid when the scattered field is significantly weaker than the incident field and multiple scattering can be ignored. The first Born approximation includes only single scattering. Higher order approximations correspond to the incident field being scattered twice, and so on.

As an alternative to the Born approximation, the total field $\psi(x)$ may be expressed as the product of the incident field $\psi_i(x)$ and a perturbation term:

$$\psi(x) = \psi_i(x) e^{i\phi_e(x)}$$  \hspace{1cm} (5)

where $\phi_e(x)$ is the complex phase. A series expression for $\phi_e(x)$ can be generated where the first term in the expansion is referred to as the (first) Rydov approximation:

$$\phi_e(x) \approx \psi_s(x) / \psi_i(x)$$  \hspace{1cm} (6)

where $\psi_i(x)$ is given by Eq. (4). The Rydov solution describes how the wave fronts of the incident wave are distorted as they go through the object. If the complex phase is small, the exponential in Eq. (5) can also be expanded in a series and the Born and Rydov solutions for the total field are equivalent to second order.

It is generally believed that the Born approximation is most valid for large-angle scattering while the Rydov solution is most valid in forward, line-of-sight propagation (the usual tomographic configuration). Which formalism should be adopted for numerical implementation? There is an important practical problem associated with inversion algorithms based on the Rydov approximation. In the Born approximation, the scattered field is obtained from the measured total field by a simple subtraction of the known incident wave. In the Rydov approximation, the complex phase is obtained by taking a logarithm, which is not straightforward since the logarithm of a complex argument is multivalued and phase unwrapping is notoriously difficult. In most cases, the assumption is that the scattering is sufficiently weak that either the Born or the Rydov approximation adequately models the forward scattering process. Inversion algorithms are therefore usually based on the Born approximation with the understanding that the Rydov solution might be better if the phase can be correctly unwrapped.
3.2 Fourier Diffraction Theorem

In direct analogy with straight-ray tomography, diffraction tomography inversions are usually based on the Fourier diffraction theorem, illustrated in Fig. 8. An object characterized by scattering function \( F(x, z) \) is insonified by a plane wave propagating in direction \( \phi \). The total field is measured along a linear receiving array orthogonal to the incident field direction, and the scattered component \( \psi_s(\xi) \) is obtained by subtracting the known incident wave [Fig. 8(a)]. The theorem states that the one-dimensional Fourier transform of the scattered field maps onto a semicircle in the two-dimensional Fourier space of the object [Fig. 8(b)]. We observe that the radius of the semicircle is the wave number \( k \) of the probing wave. This result can be proved by substituting an appropriate integral representation of the Green's function into Eq. (4) and taking the transform of the scattered field (DeVane, 1982).

By rotating either the measurement system or the object, the scattered field can be recorded at other angles. Rotating the insonification vector corresponds in Fourier space to rotating the semicircle, much in the same way as a line in Fourier space is rotated in the nondiffracting case. If we were to take views at all directions over 360°, we would cover all spatial frequencies out to a cutoff wave number \( \sqrt{2k} \). Higher spatial frequencies cannot be recovered from the forward scattered field, so it is only possible to recover a low-pass filtered version of the object. Physically the low-pass filtering constraint corresponds to the fact that we cannot see details of the object much smaller than the wavelength of the incident wave. Consequently, we can recover only a smoothed version of the original object.

As the frequency of the probing wave is increased, the wavelength becomes smaller and we can therefore recover more of the fine detail in the object. This can be observed from Fig. 8(b). Raising the frequency increases the wave number and hence the radius of the semicircle. In the high frequency limit, Fig. 8(b) reduces to Fig. 5(b) and diffraction can be neglected.

3.3 Inversion Algorithms

Having formulated the scattering problem, we now consider techniques for inverting the scattered field to determine the scattering function. Inversion is more complicated when diffraction effects are considered; the measured quantity \( \psi_s \) is related to the unknown scattering function by a volume integral containing a Green's function [Eq. (4)] rather than a simple line integral [Eq. (1)].

Given the Fourier diffraction theorem, there are two basic techniques for inverting the measured scattered fields and reconstructing the two-dimensional object. The first essentially reduces to interpolation in the frequency domain, while the second involves interpolation in the spatial domain.

The first and most obvious reconstruction technique makes use of the discrete Fourier transform. The scattered field, which is measured along a linear recording surface, is uniformly sampled. The continuous transform of the field is then efficiently approximated using the fast Fourier transform (FFT). The discrete transform maps onto sampled arcs in the two-dimensional discrete Fourier space of the object. We can then, in principle, perform an inverse transform into real space to yield the reconstruction. The inverse transform is best achieved by a two-dimensional FFT which requires that the data be distributed on a uniform grid. The reconstruction problem therefore becomes one of interpolating from sampled arcs onto a uniform grid. Bilinear interpolation appears to work satisfactorily in some cases (Pan and Kak, 1983).

The second reconstruction approach operates entirely in the spatial domain. The filtered backprojection method used in conventional straight-ray tomography is modified to include an additional depth-dependent filter (DeVane, 1982). The backpropagation filter attempts to model the acoustic wave propagation within a finite depth interval. An example of this reconstruction technique was given in Fig. 1.

The Fourier diffraction theorem suggests another possible approach to the inversion problem. As shown in Fig. 8(b), the radius of the mapped semicircle is the wave number \( k \) of the probing wave. Recalling that the wave number is proportional to the temporal frequency, this implies that if the object were probed by a plane wave with a wide temporal
bandwidth, additional information would be recovered in spatial Fourier space. It can be shown that there is a duality between probing a weak scatterer from many directions using a single-frequency wave and probing at a single direction using a wideband pulse (Porter, 1989). It should be noted, however, that the validity of the Born approximation becomes particularly questionable at high frequencies.

The Fourier diffraction theorem is easily extended to three dimensions. The two-dimensional transform of the scattered field measured along a plane maps onto a spherical shell in the three-dimensional Fourier space of the object (Wolf, 1969). In three dimensions, the incident field will be scattered out of the plane containing the insonification vector. Consequently, unlike straight-ray tomography, three-dimensional reconstruction cannot be accomplished by a succession of independent two-dimensional problems. Optimal inversion requires that the object be insonified at all possible angles over a complete sphere. In practice, this substantially increases both the number of views needed and the computational effort required to invert the data.

### 3.4 Examples of Diffraction Tomography

Diffraction tomography has yet to find widespread clinical applications. The fact that the two-dimensional algorithms of the preceding section are not easily extended to full three-dimensional inversion has limited their usefulness. Another major difficulty is the weak-scattering assumption; the measured field must obey either the Born or Rytov approximation. In the medical application, bones are not weak scatterers.

The clinical application of diffraction tomography with the greatest potential is imaging the breast. As we saw in the previous section, cancerous lesions can often be characterized as regions with relatively small perturbations in the acoustic index of refraction (Schreiman et al., 1984); it is therefore hoped that the forward scattering can be accurately modeled by weak scattering.

In cases where the scattering is strong, Eq. (4) cannot be linearized and diffraction tomography does not yield a quantitative reconstruction of an object’s acoustic parameters. Rather, it gives the sources induced in the object which depend on the pressure field within. The qualitative images that are produced, however, may reveal features of the object that could not be resolved by nontomographic techniques.

Duchêne et al. (1988) have reported on qualitative experimental reconstruction of cylindrical targets. They used an acoustic frequency and corresponding wavelength of 2 MHz and \( \lambda_0 = 0.75 \text{ mm} \), respectively. The targets were a rubber pipe and a nylon string. The inside and outside diameters of the pipe were 2 and 5 mm (2.7\( \lambda_0 \) and 6.8\( \lambda_0 \)), and the sound speed was 1560 m/s. The nylon string with a diameter of 0.6 mm (0.8\( \lambda_0 \)) was highly refractive with a sound speed of 2600 m/s. A line receiver with 80 individual elements 0.7\( \lambda_0 \) apart was used as the receiving array, with coupling between elements measured and taken into account in the processing. The magnitude of the complex reconstruction of a single pipe and string is shown in Fig. 9. This image was built up from 32 views. It appears to be qualitatively correct except for the bright spot in the pipe wall; this can be attributed to the nonoptimal element spacing. These authors employed the same technique to image the heart of a rabbit (size \( \approx 24 \lambda_0 \)), but the result is qualitative at best. Much remains to be done before high quality reconstructions can be realized.

Another potential application of diffraction tomography is imaging ocean microstructure, the perturbations in the sound speed field on scales less than a meter. Since the relative variance of these perturbations is on the order of \( 10^{-8} \), the forward scattering problem is well modeled by either the Born
4. RAYS IN A WAVEGUIDE: OCEAN TOMOGRAPHY

Observing the ocean is an exceedingly challenging task because of the wide range of oceanic time and space scales. Spatial scales range from millimeters to megameters, while temporal variability ranges from seconds to years. Oceanographic measurements until recently have typically been point measurements in space and/or time. Aliasing because of undersampling has been the nemesis of oceanographers since the founding of the discipline over a hundred years ago. Satellite sensors will help a great deal in that they offer very rapid measurements over essentially the entire surface of the Earth, but they do not provide information on the interior of the ocean. However, the ocean is transparent to sound, and oceanic processes change the sound speed field by several percent. These facts led to the idea of ocean acoustic tomography, a method for using sound to probe the three-dimensional structure of the oceans.

This concept was formalized by Munk and Wunsch (1979). Their idea, reminiscent of medical x-ray tomography, was to transmit sound between instruments in a moored array over ranges of hundreds of kilometers. A mesoscale eddy or some other ocean feature would affect the travel times along some transmission paths but not others (Fig. 10). Because of practical limits on the number of instruments involved and thus the number of data, one must deal with many individual transmission paths rather than a projection made up of many paths. Geophysical inverse theory is used to sort out the magnitudes and locations of the perturbations. This measurement technique has many attractive features. Compared with conventional sampling meth-

FIG. 10. Acoustic transceivers send and receive pulses along many paths. Inhomogeneities such as eddies and fronts alter the sound speed and thus change the travel times (from Spindel, 1986; @American Society of Mechanical Engineers).

ods, relatively few instruments are needed to sample a large volume because the number of data grows quadratically with the number of instruments, rather than linearly as with point measurements. Furthermore, because of the ocean sound speed channel (discussed in the next section), sound travels along many different paths sampling different depths, so information on the depth structure can be obtained from a single source-receiver pair. Because the measurement inherently averages the ocean along the path the sound takes, small-scale ocean processes are averaged out, thus improving the resolution of the larger scale features. Also, because sound travels faster with a current than against one, differential travel time is a measure of the water velocity along the path.

4.1 Forward Problem

Before one can use measured travel times to obtain maps of sound speed (roughly proportional to temperature) or currents, one must be able to predict the travel times for a given sound speed field. The feature that most distinguishes ocean tomography from other kinds of tomography is the ocean sound channel. The speed of sound (inversely proportional to the index of refraction) typically decreases from the surface as the water temperature decreases, but then increases due to the competing effects of pressure (see Acous-
FIG. 11. (Left) Reference sound speed profile $C_0(x)$. This range-averaged profile was obtained by smoothing fairly deep historical data with profiles obtained every 20 km to 1800 m depth. (Right) Corresponding eigenrays. Note the ray groups with shallow and deep upper turning points as well as the axial ray (from Howe et al., 1987).

The effect is to produce a waveguide; sound in the waveguide or channel is constantly being refracted back toward the axis of the channel and travels along many quasisinusoidal rays (Fig. 11). Typical ray “wavelengths” are 50 km. The sound leaves an omnidirectional sound source in all directions; only sound leaving at specific launch angles along eigenrays (Fermat paths) will intersect a receiver. The energy from a single transmitted pulse will be split up among the various rays and will arrive at the receiver at different times, not only because the different rays have different lengths but because they go through regions of the ocean with different sound speed. A typical arrival pattern is shown in Fig. 12.

The success of the method depends on several essential features of ocean acoustic propagation. Separate ray arrivals must be resolved, the arrival pulse must be identified with a known ray path, and the ray paths must be stable over the duration of the experiment. A 1979 experiment verified that arrival patterns remained stable over long periods of time; mesoscale and other oceanic processes merely distorted but did not confuse the pattern (Spiesberger et al., 1980). The small changes in the travel times of the ray arrival peaks are the data for tomography. Typical changes in travel time due to sound speed perturbations and currents are 100 and 10 ms, respectively.

In the geometric optics approximation the travel time along a ray path $\Gamma_i$ in the presence of a current $u(x, t)$ is

$$T_i(t) = \int_{\Gamma_i} \frac{ds}{C(x, t) + u(x, t) \cdot \tau}$$  \hspace{1cm} (7)

where $\tau$ is a unit vector tangent to the ray and $s$ is arc length. The sign of $u \cdot \tau$ depends on the direction of propagation, and the travel times in opposite directions differ because of the effects of currents. The ray paths $\Gamma_i$ are obtained using a numerical eigenray code. Equation (7) is analogous to Eq. (1) but is for one ray rather than a multitude of parallel paths.

The travel-time path integral [Eq. (7)] is nonlinear in propagation velocity. Defining a reference sound speed field $C_0(x)$, one has

$$C(x, t) = C_0(x) + \delta C(x, t),$$  \hspace{1cm} (8)

where the perturbation sound speed $\delta C(x, t) \ll C_0(x)$. Substituting in Eq. (7), forming perturbation travel times, and linearizing gives
The measured arrival pattern is an average for day 217. The arrival pattern at a hydrophone (vertical, units as time) on the measured arrival pattern for data on the WQD model (Brown, 1982) and for wave propagation along the WQD model (Brown, 1982) from the source, there is a total number of wave travel times. The measured arrival pattern is an average for day 217. The arrival pattern at a hydrophone (vertical, units as time) on the measured arrival pattern for data on the WQD model (Brown, 1982) and for wave propagation along the WQD model (Brown, 1982) from the source, there is a total number of wave travel times. The measured arrival pattern is an average for day 217. The arrival pattern at a hydrophone (vertical, units as time) on the measured arrival pattern for data on the WQD model (Brown, 1982) and for wave propagation along the WQD model (Brown, 1982) from the source, there is a total number of wave travel times. The measured arrival pattern is an average for day 217. The arrival pattern at a hydrophone (vertical, units as time) on the measured arrival pattern for data on the WQD model (Brown, 1982) and for wave propagation along the WQD model (Brown, 1982) from the source, there is a total number of wave travel times.
\[ s_i = \frac{1}{2} (\delta T_i^+ + \delta T_i^-) = - \int_{r_{\text{out}}} \frac{\delta C(x, t)}{C_0^2(x)} ds, \tag{12} \]

\[ d_i = \frac{1}{2} (\delta T_i^+ - \delta T_i^-) = - \int_{r_{\text{out}}} \frac{u(x, t) \cdot \mathbf{r}}{C_0^2(x)} ds. \tag{13} \]

The problem has separated: \( s_i \) is linearly related to the sound speed perturbation, while \( d_i \) is linearly related to the water velocity \( \mathbf{u} \cdot \mathbf{r} \) along the ray path.

### 4.2 Inverse Problem

Equations (12) and (13) constitute the acoustic forward problem: Continuous \( \delta C \) and \( u \) fields are used to predict a finite data set \( s_i \) and \( d_i \). The corresponding inverse problem uses the measured data to estimate the continuous fields. The problem is an underdetermined one because values of the \( \delta C \) and \( u \) fields must be specified at all \( x \) using only a finite number of data. To proceed, the fields \( \delta C \) and \( u \) are parametrized with a finite number of discrete parameters by a model that is oceanographically meaningful. This procedure has three advantages: The parametrization can be very efficient, the inverse solution can become more accurate because the model injects our oceanographic knowledge concerning permissible structure of the fields (reducing the degrees of freedom), and the procedure can be used to evaluate data-model consistency.

Current models are represented by a linear combination of basis functions describing the horizontal \((x, y)\) and vertical \((z)\) structures,

\[ \delta C(x, y, z) = \sum_{i, j, k} a_{ijk} X_i(x) Y_j(y) Z_k(z). \tag{14} \]

The coefficients \( a_{ijk} \) are the model parameters we are trying to determine. The horizontal dependence is often taken to be a truncated Fourier series or, even more simply, pixels. The functions describing the vertical dependence \( Z_k(z) \) are chosen to efficiently parametrize expected variations in the ocean. Typically they are functions based on oceanographic theory or data which describe the dominant modes of variability (see OCEANOGRAPHY).

To obtain estimates of the model parameters and thus the sound speed anomaly from the measured travel times, the "stochastic inverse" of geophysics (see GEOPHYSICS) is used (Aki and Richards, 1980). The method assumes that an \( a \) priori estimate of the mean and covariance of the unknown field exists, and constructs an estimator that minimizes the square of the difference between the estimate of the unknown field and the true field. In the overdetermined case, this reduces to the weighted least squares fit where the sum of the squares of the data residuals (weighted by the inverse of the data error) is minimized and the \( a \) priori information is not necessary. In the mixed or underdetermined case, the problem is constrained by adding the \( a \) priori information about the model parameters. In the absence of any data, the best estimate of the field is simply the \( a \) priori estimate of the mean, and the associated error is just the \( a \) priori estimate of the covariance. The stochastic inverse provides a rational way of combining measurements and \( a \) priori information. Such a method is almost always necessary in geophysical, data poor, situations.

The generalized forward problem can be written

\[ \mathbf{d} = \mathbf{Gm} + \mathbf{n}. \tag{15} \]

The vector \( \mathbf{d} \) contains the data, for instance the \( s_i \) in Eq. (12), and \( \mathbf{m} \) contains the "true" model parameters [the \( a_{ijk} \) in Eq. (14)]. \( \mathbf{G} \) is the matrix of partial derivatives \( G_{ij} = \partial d_i / \partial m_j \) relating the model parameters to the data, obtained by substituting the model Eq. (14) into Eq. (12) and differentiating, while measurement noise and uncertainty in the forward problem physics and model are incorporated in the noise vector \( \mathbf{n} \). The stochastic inverse operator is given by (Aki and Richards, 1980)

\[ \mathbf{L} = \mathbf{W} \mathbf{G}^T (\mathbf{G} \mathbf{W} \mathbf{G}^T + \mathbf{E})^{-1}. \tag{16} \]

The matrices \( \mathbf{W} \) and \( \mathbf{E} \) are the model and data error covariances, respectively. The estimated model parameters \( \mathbf{m} \) are given by

\[ \mathbf{m} = \mathbf{Ld}. \tag{17} \]

The model parameters are a linear combination of the data.

In a mixed or underdetermined problem it is important to have a measure of how well each model parameter is resolved by the data. The resolution matrix \( \mathbf{R} \) is defined by

\[ \mathbf{m} = \mathbf{R} \mathbf{m}. \tag{18} \]

The \( \mathbf{G} \) matrix along with the covariance matrices enter into the calculation of the resolu-
tion matrix (Menke, 1989). The resolution matrix shows how well the combined measuring system and the reconstruction method are able to determine each model parameter. Ideally $\mathbf{R}$ is the identity matrix so that the estimated model parameters $\hat{\mathbf{m}}$ equal the true model parameters $\mathbf{m}$. Unfortunately, this is rarely the case.

If a Fourier spectral model is being used, the resolution matrix will tell how well each Fourier coefficient is resolved, i.e., how well Fourier space is filled, in a way analogous to the Projection Slice theorem. If a simple pixel model is used, one can directly obtain the impulse response, i.e., how well a unit perturbation in one pixel is reconstructed, and how the reconstruction depends on the values in all the other pixels.

When data are time dependent, one can use a Kalman filter to assimilate new data. The field at the new time is first estimated from the old field using either a rigorous physical model, such as a three-dimensional, primitive equation numerical model, or ad hoc methods such as letting the field decay to the a priori values with some characteristic time scale. The error covariance is estimated in a similar fashion. The inverse is then performed with these estimates of the mean and covariance of the field, producing a new estimate of the mean and error covariance.

Over long ranges it is no longer valid to assume the ray paths in the reference ocean are "close" to the ray paths in the actual ocean. This introduces a nonlinearity, the degree of nonlinearity being proportional to range and the average eddy energy along the path (Spiesberger, 1985; Munk and Wunsch, 1987). For a horizontal array of instruments with many crossing paths in the horizontal, the first inversion should be able to place the eddies in approximately the right location, at least within a fraction of a ray wavelength (implying some resolution at all wave numbers). Simple iteration should converge to the correct solution. For a single-slice experiment, there is some information in the travel times about range dependence, but it is not clear whether there will be enough information for an iteration to converge.

In the source localization problem (e.g., finding submarines), one takes the entire measured wave form over a large spatial array and solves for source position; this is called matched field processing and incorporates aspects of matched filtering, backpropagation, and holography (Baggeroer et al., 1988). This is done by assuming various source positions and predicting the wave form at the array until the difference between the predicted and measured wave forms is small. This technique is in its infancy and will most likely have to include sound speed variability. In the typical ocean tomography case, the instrument positions are usually known to within a few tens of meters and this position residual can be included as a model parameter in the (linear) inverse. In the localization problem, the position is not known to even a convergence zone, making the problem very nonlinear with many local solutions. The links between the tomography and localization problems are being actively pursued.

### 4.3 Examples of Ocean Tomography

An ocean acoustic tomography system requires instruments capable of sending and receiving acoustic pulses whose duration is short enough to resolve separate ray arrivals, that are precisely timed to allow absolute measurement of travel time, and are strong enough to be received above the ambient noise. A short duration pulse implies a broad bandwidth signal; present sources have a bandwidth approaching 100 Hz which gives a pulse width of 10 ms. Precision time keeping (1 ms/yr) is accomplished by using rubidium frequency standards in the underwater instruments. Even the loudest acoustic sources available (193 dB re 1 $\mu$Pa at 1 m) do not produce enough energy in a 10 ms pulse to be heard at any significant distance. The remedy has been to transmit a long, coded signal, spreading the signal energy out in time, and then in the signal processing to recombine the energy to make it appear as if it all had been sent in one loud pulse. The signals sent are pseudorandom, phase-modulated sequences; the processing is called replica cross correlation, which is closely related to matched filtering. (The Hadamard transform is used to do this efficiently.) A signal processing gain of 35 dB is typical.

One of the tomography transceivers presently being used is shown in Fig. 13. The instruments are deployed on single-point, subsurface oceanographic moorings. Although the moorings are designed to be quite stiff,
how nearly reciprocal the paths in each direction are; if the paths are exactly reciprocal, then the effects of internal waves on the one-way travel times (3–10 ms rms) will cancel. In the 1983 reciprocal transmission experiment over 300 km, the reciprocal noise variance was smaller than the one-way noise variance by a factor of 10.

More recently, three tomography transceivers were deployed in the northeast Pacific in a 1 Mm triangle for four months. The primary purpose of the experiment was to measure large-scale gyre vorticity or swirl: By measuring the current velocity along each leg of the triangle using reciprocal transmissions, one can calculate the circulation around the triangle, \( \oint \mathbf{u} \cdot d\mathbf{l} \). This is in turn proportional to the average vorticity by Stokes's theorem. The vorticity is a desirable quantity because it is the basis for much of theoretical oceanography, yet it has never been measured directly on large scales. This experiment exploits the integrating (projection) nature of the method to filter out the mesoscale features, leaving the gyre-scale features of interest.

There are several ocean tomography projects in progress. One involves six moorings deployed in September 1988 in the Greenland Sea for one year. This area far to the north is interesting because it is thought that very cold, dense water forms during the winter in small-scale overturning events, then sinks, and finally spreads throughout the world ocean. This has never been directly observed; it is hoped that the tomography data will show indications of these events. A complicating feature of acoustic propagation in polar waters is that the sound speed monotonically increases with depth producing a surface-bounded waveguide; all rays are surface reflected. Acoustic energy propagating within several hundred meters of the surface is not well modeled by ray theory. The dual of rays, normal modes, is necessary to describe the propagation.

The moorings in the Greenland Sea are also being used for engineering tests of moving ship tomography (Cornuelle et al., 1989). A simulated geometry is shown in Fig. 16. The idea in this application is to obtain a “snapshot” of the ocean by circumnavigating the moored array, dipping a hydrophone array every 4 h or 70 km to listen to the sources, and thereby building up a net of many rays. This is an obvious analogy to x-ray tomogra-
phy. For this technique to succeed, it is necessary to locate the receiving hydrophones to better than 10 m accuracy. Two acoustic tracking systems are being used in conjunction with the NAVSTAR Global Positioning System, a very long-baseline system using satellites.

Munk and Forbes (1989) have proposed placing a source off Heard Island south of Australia and listening to it as far away as Bermuda and California. This idea is motivated by the reanalysis of a 1960 experiment in which a 140-kg explosive charge was detonated off Perth, Australia, and heard in Bermuda some four hours later. Such long-range transmissions will average out much of the ocean variability, and it is possible that changes in travel time will be proportional to global-scale changes in ocean temperature. Munk and Forbes calculate that a 4 mdeg C/yr change corresponds to a 100 ms/yr change in travel time for a Heard Island–California path. A major unanswered question is whether the forward problem can be adequately solved so that a quantitative inversion can be made. Ultimately, a tomographic network is envisioned where sources and receivers will be distributed around the world, continually monitoring the global ocean.

5. CONCLUDING REMARKS

The ultimate physical limit to spatial resolution in any tomography experiment is the acoustic wavelength. The image quality and accuracy achieved in a particular experiment will depend on a multitude of factors including the number of transducers, the number of views, the size of the viewing aperture, and the signal-to-noise ratio. All of these are controlled to a large extent by practical considerations such as geometry, signal processing, and cost (Schueler et al., 1984).
However, the accuracy of any quantitative reconstruction will be directly related to the quality of the forward-problem solution. Any inaccuracies in the forward problem will appear in the inversion or reconstruction. This is the major problem facing diffraction tomography with strong scattering. The Born and Rytov approximations are often not valid, implying that higher order scattering needs to be taken into account as well as three-dimensional effects. This will make the problem nonlinear and thus even more difficult. With desktop minisupercomputers becoming more common, brute force numerical approaches may be appropriate.

Straight-ray tomography is relatively easy to perform, but again the accuracy of the forward problem must be checked. For instance, a ray trace through the reconstructed sound-speed field should be done to see if the differences in travel times along the curved rays are significantly different from those along the straight rays, or whether diffraction effects are important.

Ocean tomography is just evolving from the testing and demonstration stage and is at the point where useful oceanographic measurements are starting to be made. Current research is focused on improving horizontal resolution by making use of the special nature of the ocean waveguide and the ocean structure (Cornuelle and Howe, 1987), recasting the forward/inverse problems in terms of acoustic normal modes, creating synthetic apertures with moving instruments, and using long range transmissions aimed at basin and global monitoring.

Many difficult problems remain to be solved before acoustic tomography becomes a common tool of scientists and engineers. It is clear, though, that acoustic methods can yield information that is either unobtainable any
FIG. 16. Plan view of the ray geometry in a simulated moving-ship tomography experiment. Four sources are located on a 350-km-radius circle about the center of a 1000-km by 1000-km square. Receivers are located at approximately 70-km intervals around the square. In this simulation the ocean is assumed to be two dimensional, with no vertical structure; straight-line ray paths can therefore be used in the calculation. The inversion procedure (using realistic travel-time and positioning errors) leads to mapping which accounts for 95% of the mesoscale variance. Subsequent 3-D simulations with vertical multipaths lead to similar mapping errors. In this preliminary simulation all receptions are assumed to be simultaneous. The actual experiment will be conducted by lowering a receiver from a ship at roughly 70-km intervals as it transits around the square. This involves "nowcasting" in a 4-D (space + time) environment (from Munk and Worcester, 1988; ©1988, The Oceanography Society).

other way or complementary to results from other methods.

GLOSSARY

**Born Approximation:** Linearizing approximation in scattering theory. Approximates unknown total field in a scatterer by the known incident field.

**Diffraction:** Deviation of energy from straight ray paths due to effects other than reflection or refraction. Occurs when size of irregularities in a medium become comparable to the wavelength of the probing wave.

**Forward Problem:** Predicting data given the parameters which affect the data.

**Fourier Diffraction Theorem:** Fundamental theorem of diffraction tomography. Fourier-domain relationship between measured data and unknown scatterer.

**Inverse Problem:** Determining the parameters which affect measured data.

Ray: A curve whose tangent at any point lies in the direction of propagation of a wave.

Refraction: The bending of a wave as it passes through regions with varying wave velocities or indexes of refraction.

Tomography: Imaging procedure that uses multiple views to produce quantitative cross-sectional reconstructions of an object.

Waveguide: A physical situation which constrains or guides the propagation of waves by reflection off solid boundaries and/or by refraction within the medium.

List of Works Cited


Further Reading


