Multiple Receivers in Single Vertical Slice
Ocean Acoustic Tomography Experiments

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Range-dependent information about the ocean can be obtained from a vertical slice tomography experiment with a single source-receiver pair (Howe et al., 1987). For this situation we consider the improvement to the estimated fields of sound speed (temperature) and current velocity possible from receiving hydrophones widely spaced in the vertical at each end of the slice. Simulations were carried out varying the number of receivers (1–5), the range, and the travel time data error. The ocean was represented by a quasi-geostrophic spectral model with a Gaussian correlation length scale of 100 km. Results are as follows: (1) the range-dependent sound speed field is well resolved at 300 km with additional receivers but poorly resolved at 1000 km, (2) the error in the range-averaged sound speed is reduced substantially by adding more receivers, and (3) errors in the range-dependent and range-averaged current velocity are unaffected by adding receivers.

1. INTRODUCTION

In this paper we investigate how ocean acoustic tomography experiments can be improved by using additional receivers widely spaced in the vertical. The simulated experimental configuration consists of a transmitter and a variable number of receiving hydrophones at each end of a single vertical slice or section in the ocean. Estimated errors in the fields of perturbation sound speed $\delta C(x,z)$ (temperature) and water velocity $u(x,z)$ are used to evaluate the performance of a particular configuration as a function of the number of receivers (1, 3, and 5), the range (300, 600, and 1000 km), and the travel time data error.

The Reciprocal Transmission Experiment of 1983 (RTE83) was a vertical slice experiment over a 300 km range with single sources and receivers at each end [Howe et al., 1987]. An unexpected result of this experiment was that a significant fraction of the a priori range-dependent variance in the sound speed field could be resolved. This immediately raises the questions of whether a better experimental configuration exists for this measurement, and over what ranges can one expect to resolve range-dependent features? Calculations for the RTE83 case showed that most of the remaining error in the sound speed field was due not to data error but rather to lack of resolution, because significant parts of the ocean remained unsampled by the acoustic rays used. B. Cornuelle suggested using additional receivers, and thus rays and data, as a possible way of improving resolution. This was the motivation behind this work.

Ideally, one would want to use additional transceivers and thereby improve not only the sound speed field estimate but also the current velocity field estimate. In practice though, acoustic sources are not easy to add because they are large, heavy, and expensive. Receiving hydrophones, on the other hand, are easy to add for just the opposite reasons.

The plan of the paper is as follows. Section 2 contains a brief review of ocean acoustic tomography including the forward and inverse problems and the ocean model used. Section 3 describes the a priori statistics assumed for the ocean model and the different cases simulated. Following that are the results of the simulations and a discussion.

2. BACKGROUND

Acoustic Forward Problem

The linearized forward problem in acoustic tomography for a single slice can be written

$$\delta T_i^z = T_i^z - T_0^z = -\int_0^R \frac{dx}{C_0^2(z) \cos \theta} \left[ \delta C(x,z) \pm u(x,z) \right]$$

(1)

where

$$T_0^z = \int_0^R \frac{dx}{C_0(z) \cos \theta}$$

is the reference travel time along ray $i$ and $\delta T_i^z$ is the perturbation travel time in the $\pm z$ direction. The total sound speed is

$$C(x,z) = C_0(z) + \delta C(x,z) \quad \delta C << C_0$$

$u(x,z)$ is the component of the water velocity in the positive $x$ direction, $z = z_0(x)$ is the trajectory of the $i$th geometric ray path in the reference sound speed field $C_0(z)$, and $\theta$ is the angle the ray makes with the horizontal. The range $R$ between source and receiver is assumed to be known exactly.

We use the reference sound speed profile $C_0(z)$ from Howe et al. [1987] measured in August 1983 at 32°N, 70°W to the southwest of Bermuda (Figure 1a). Twelve geometric rays are traced between two transceivers at 1350-m depth and separated by 300 km (Figure 1b). Because of the near-constant sound speed region between 200 and 600 m, the rays fall into two groups: ones with shallow upper turning depths (~100 m) and ones with deep upper turning depths (~600 m). The arrival pattern predicted using the WKBJ method of Brown [1982] is
shown in Figure 1c. The shallow turning rays arrive early and the deeper ones later.

As can be seen in Figure 1b there are regions of the ocean which are left unsampled. Placing two additional receivers at the upper and lower turning depths of any one ray results in the maximum horizontal shift of the ray (Figure 2). The horizontal separation between the two new ray paths grows from zero at the source to a value equal to the single loop range at the receiver, between 10 km and 40 km depending on the ray. Additional receivers in between give little new, oceanographically independent information.

Fig. 2. Additional ray paths resulting from adding two receivers near the upper and lower turning depths of only the -14 ray. Rays are identified as $\pm n$, where $n$ is the total number of upper and lower loops and the $\pm$ sign corresponds to a $\pm$ launch angle at the source.
Because the rays in each of the two groups for this particular profile are so alike, only four additional receivers provide nearly complete coverage. The receiver depths were chosen subjectively to most completely fill in the section. In Figure 3a, rays are plotted from a single source at 1350 m on the left to five receivers at 600, 1000, 1350, 2400, and 3400 m on the right. The coverage of the slice on the right side is increased. With multiple receivers at both ends of the slice there will be twice as many rays as in Figure 3a, and the section will be nearly completely filled in by rays; the corresponding figure could be constructed by superposing the left-to-right mirror image of Figure 3a on itself. The WKBJ-predicted arrival patterns for the five receivers are shown in Figure 3b. Note the splitting of the even-numbered arrivals and the merging of odd- and even-numbered arrivals as the vertical distance from the sound channel axis is increased. For this work we elected to use the 12 rays identified in the figure because they were the ones we were able to use in analyzing the RTE83 data [Howe et al., 1987]. As one goes to larger ranges, the number of arrivals increases by the ratio $R_{\text{new}}/300$ km, as do the value of the ray identifier (the total number of upper and lower ray loops) and the time spread of the total pattern. Thus at 600 km, 24 rays are used, and the total arrival pattern has spread from 1.5 to 3 s long. The ray identifiers have changed from 8 through 15 to 16 through 30.

Ocean Model

We represent the ocean by the quasi-geostrophic spectral model described by Cornuelle [1983] and Howe et al. [1987]. The vertical structure is specified by Rossby wave modes $Z_i(z)$ and the horizontal structure by sines and cosines,

$$\delta C(x, z) = \sum_{\nu} \left[ A_{\nu}^C \cos \frac{2\pi \nu x}{L} + B_{\nu}^C \sin \frac{2\pi \nu x}{L} \right] Z_i^C(z)$$

and similarly for $u(x, z)$. By ordering the $A_{\nu}^C$ and $B_{\nu}^C$ into a vector $m^C$ with $M^C$ components, this expression can be rewritten

$$\delta C(x, z) = \sum_{\nu} m_{\nu}^C X(\nu) Z_i^C(z) \quad k = jN_{\nu}^C + i$$

$$\delta C(x, z) = \sum_{k-1} m_{k}^C P_C(x, z)$$

The $X_i$ are the $N_{\nu}$ functions

$$\left[ 1, 0, \cos \frac{2\pi x}{L}, \sin \frac{2\pi x}{L}, \cos \frac{4\pi x}{L}, \ldots, \sin \frac{2\pi N_{\nu} x}{L} \right]$$

where $N_{\nu} = N_{\nu} - 1$. The expansion domain $L$ is chosen to be twice the experimental domain to avoid periodicity effects.
TABLE 1. Travel Time Data Errors (in ms)²

<table>
<thead>
<tr>
<th></th>
<th>300</th>
<th>600</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>Internal wave displacements</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Internal wave velocities</td>
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<td>0.4</td>
<td>0.4</td>
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<tr>
<td>Residual mooring motion</td>
<td>14</td>
<td>3.5</td>
<td>0</td>
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<tr>
<td>Ambient acoustic noise</td>
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<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>Interpolation error</td>
<td>0.3</td>
<td>0.03</td>
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</tr>
<tr>
<td>N samples per day</td>
<td>12</td>
<td>144</td>
<td>576</td>
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<tr>
<td>Nonlinearity</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>Clock error</td>
<td>0.1</td>
<td>0.025</td>
<td>0</td>
</tr>
<tr>
<td>Net error variance, (ms)²</td>
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<tr>
<td>One-way</td>
<td>3.0</td>
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<tr>
<td>Reciprocal</td>
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<td>0.028</td>
<td>0.00069</td>
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<td>Net rms error, ms</td>
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<tr>
<td>One-way</td>
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<tr>
<td>Reciprocal</td>
<td>0.43</td>
<td>0.17</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Inverse Problem

Equation (3) for \( \delta T_i(x, z) \) and the corresponding one for \( u(x, z) \) can be substituted in (1) to give

\[
\delta T_i^\pm = - \int_0^R \frac{dx}{C_i^\pm(x, z)} \left[ \frac{1}{\cos \theta} \sum_{j=1}^{M} m_j^C P_j^C(x, z) \right] \\
\pm \sum_{j=1}^{M} m_j^P P_j^P(x, z) \right] = \sum_{j=1}^{M} m_j G_{ij}
\]

where \( \mathbf{m} = [m_C \mathbf{m}^P] \) and \( G_{ij} = \delta \frac{\partial T_i}{\partial m_j} \). In matrix notation (with data \( d_i = \delta T_i \))

\[
\mathbf{d} = \mathbf{Gm} + \boldsymbol{\epsilon}
\]

where \( \boldsymbol{\epsilon} \) represents data error and inadequacies in the model.

We wish to obtain estimates \( \hat{\mathbf{m}} \) of the true model parameters \( \mathbf{m} \). In general, the problem is mixed or under-determined; there are more model parameters than data. To obtain a unique inverse of \( \mathbf{G} \), we use the weighted, damped, least squares procedure which minimizes the Euclidian norm of the (weighted) data residuals \( \| \mathbf{d} - \hat{\mathbf{d}} \|_2 \) and the estimated (weighted) model parameters \( \| \hat{\mathbf{m}} - \mathbf{m} \|_2 \). The method is outlined by Lawson and Hanson [1974] and by Aki and Richards [1980].

This method produces the generalized inverse \( \mathbf{G}^{-\#} \) so that

\[
\hat{\mathbf{m}} = \mathbf{G}^{-\#} \mathbf{d}
\]

with the model error covariance matrix

\[
\hat{\mathbf{C}} = \left( \mathbf{m} - \hat{\mathbf{m}} \right) \left( \mathbf{m} - \hat{\mathbf{m}} \right)^T
\]

The method depends on specifying the a priori model covariance matrix \( \mathbf{C} \) and the data error covariance matrix \( \mathbf{C}_e \). The former is taken to be diagonal, but the latter is not because reciprocal rays between the two transceivers at 1350 m have correlated noise. Noise due to internal wave displacements and nonlinearities present in one-way travel times cancel in the case of reciprocal transmissions.

3. Numerical Simulations

Parameter Ranges

In the simulations we varied three parameters: the number of receivers, the range, and the data error. The number of receivers used was 1, 3, and 5 at each end of the slice at depths (1350 m), (600, 1350, 2400 m), and (600, 1000, 1350, 2400, 3400 m). The ranges were 300, 600, and 1000 km. The number of rays used (and thus one-way travel time data) at each of the five receiver depths was (6, 24, 24, 22, 10, \( \Sigma = 86 \)), (12, 44, 48, 40, 12, \( \Sigma = 156 \)), and (20, 80, 80, 68, 20, \( \Sigma = 268 \)) for the three ranges, respectively.

The data error cases are a function of range with the values chosen to represent specific situations (Table 1). For 300 km, data error case 1 is the same as that used in analyzing the RTE83 data. Case 2 represents a realistic situation where equipment has been improved so mooring motion, clock, and interpolation errors are reduced. Case 3 represents the best one might ever expect: only internal wave noise is present and it is assumed (optimistically) to be independent at the high sampling rate of once every 2.5 min. Internal wave travel time variance has been assumed to grow linearly with range [Flatté and Stoughton, 1986].

At long ranges, errors in the linearization of the acoustic forward problem become significant. This nonlinearity arises because the reference ray paths are somewhat distorted by perturbations in the sound speed field, producing an apparent travel time perturbation not linearly related to the sound speed perturbation [Mercer and Booker, 1983; Spiesberger and Worcester, 1983]. The rms errors for nonlinearity were taken to be 3 ms rms and 6 ms rms for the 600-km and 1000-km ranges, respectively, based on the simulation results of Spiesberger [1985]. The 0.9-ms rms error for nonlinearity at 300 km was taken from Howe [1986] for the RTE83. The nonlinearity can be reduced by iteration (assuming the first result is in the right direc-
Fig. 4. Total depth- and range-averaged a priori variance for sound speed perturbation $\delta C$ and current velocity $u$. $RD$ stands for range-dependent, and $RA$ stands for range-averaged. The left axis is for perturbation sound speed $\delta C$, and the right axis is for current velocity $u$.

...tion) and/or by assuming it can be calculated, as was suggested by Spiesberger [1985] and Munk and Wunsch [1987]. In case 2 at 600 and 1000 km, the nonlinearity was reduced a factor of 4 to represent the first step in such a procedure. For case 3 it is assumed that a sufficient number of iterations has been performed so that there are no longer any nonlinearities. Although it is important to recognize all the individual sources of error, only the final error (see Table 1) is important for the inversion process.

A Priori Model Variances

In specifying the a priori model variances we have assumed vertical mode energy ratios of 0 : 1 : 0.2 and 1 : 0.3 : 0.1 for sound speed and current velocity, respectively. Only the first three modes are used, i.e., the barotropic and first two baroclinic modes.

To specify the horizontal structure, a homogeneous, isotropic, Gaussian covariance function with length scale $L_x = 100$ km is assumed. For mode $j$ the covariance is

$$ C_j(x) = \sigma_j^2 \exp \left[ -\frac{1}{2} \left( \frac{x}{L_x} \right)^2 \right] $$

$\sigma_j^2$ is chosen so that the total depth-averaged variance

$$ \Lambda^2 = \sum_j \sigma_j^2 \frac{1}{H} \int_0^H Z_j^2(x)dx $$

is equal to a specified value; for sound speed, $\Lambda^2 = 1.5 \text{ m}^2 \text{ s}^{-2}$ ($-0.24 \text{ °C rms}$), and for velocity, $\Lambda^2 = 60 \text{ cm}^2 \text{ s}^{-2}$. The ocean depth $H$ is 5.25 km. Actual values of the a priori model variances ($\sigma_j^2$) were determined from the spectrum corresponding to $C_j(x)$. The high wave number cutoff was specified by setting $N_F = 2, 5, \text{ and } 8$ for the three ranges, corresponding to 5, 11, and 17 horizontal degrees of freedom, respectively. With $C_j(x)$ representing the "true" ocean, more than 97% of the energy is modeled using these values of $N_F$. This particular model consisting of the truncated set of basis functions oversimplifies the ocean, but it is realistic enough to show trends in the errors.

Although these particular a priori variances and mode ratios were picked to represent the Local Dynamics Experiment (LDE) region southwest of Bermuda [Owens, 1985], the results do not depend on these absolute levels of a priori variance (and data error) because of the weighting used in the inverse procedure. The important parameter is the expected signal-to-noise ratio, the travel time variance corresponding to $\Lambda^2$ divided by the travel time error.

If we are interested in a case with $\Lambda^2$ 10 times larger than the present value, we can artificially reduce the data error by a factor of 10 and use the same results.

Error Statistics

Given an estimate $\delta C(x, z)$ of the true sound speed perturbation $\delta C(x, z)$, the estimated total error at a point $(x, z)$, is

$$ e^2(x, z) = \left( \delta C(x, z) - \delta C(x, z) \right)^2 = \sum_i \sum_j P_i^c(x, z) \hat{P}_i^c(x, z) $$

and similarly for $u$. The horizontally averaged total error is

$$ e^2(z) = \frac{1}{R} \int_0^R e^2(x, z)dx $$

and the total error averaged over the entire slice is

$$ e^2 = \frac{1}{H} \frac{1}{R} \int_0^H \int_0^R e^2(x, z)dxdz $$

We are mainly interested in the range-dependent portion of the error. Define

$$ \delta C(x, z) = \delta C_{RA}(x, z) + \delta C_{RD}(x, z) $$

with

$$ \delta C_{RA}(x, z) = \frac{1}{R} \int_0^R \delta C(x, z)dx $$

$RD$ stands for range-dependent and $RA$ stands for range-averaged (the range average over $R$ does not correspond to the range independent term in the Fourier series which is defined over $L = 2R$). With these definitions, we can calculate $\epsilon_{RD}(x, z), \epsilon_{RD}(x, z), \epsilon_{RD}(x, z), \epsilon_{RA}(x, z)$, and $\epsilon_{RA}$. To distinguish between sound speed perturbation error and current velocity error, we use the notation $c e$ and $c e$, respectively.

A Priori Error Statistics

Error estimates are equal to the a priori variances when there is no data. In this case, $e^2 = \Lambda^2$. In Figure 4, the a priori average errors $e^2, \epsilon_{RD}, \epsilon_{RA}$ are plotted versus range. The total $e^2$ is nearly constant with range (the slight increase of 3% is due to less spectral leakage for larger ranges). For larger ranges, a larger fraction of the total a priori variance is range-dependent and less is in the range-averaged fraction.

The a priori errors for $\delta C$ and $u$ as functions of depth, $e^2(z), \epsilon_{RD}(z), \epsilon_{RA}(z)$, are shown in Figure 5 for the 600 range. Most of the $\delta C$ variance is in the main thermocline, peaking at $12 \text{ m}^2 \text{ s}^{-2}$ (3.5 m s$^{-1}$ rms or 0.7°C rms). Most of the velocity variance is above the main thermocline, with a typical value of 10 cm s$^{-1}$ rms at 300 m.
Fig. 5. A priori variances as a function of depth for the 600 km range case (a) for sound speed perturbation $\delta C$ and (b) for current velocity $u$.

Most of the error results will be presented graphically as percentages of the a priori variances. The dimensional rms errors are shown in Table 2.

4. RESULTS

Sound Speed

The effect of adding receivers on the average range-dependent error $C_{\varepsilon_{RD}}$ is shown in Figure 6. For a given range and data error, $C_{\varepsilon_{RD}}$ decreases as the number of receivers increases, as may be expected. The decrease in $C_{\varepsilon_{RD}}$ is larger as the number of receivers is increased from 1 to 3 than from 3 to 5. Thus the extra two receivers do not add much more independent information about the ocean.

Fig. 6. Range-dependent error $C_{\varepsilon_{RD}}$ (percent of a priori value) as a function of the number of receivers on each mooring, for the three ranges and the three data error cases (Table 1).

Perhaps the most obvious feature in Figure 6 is the growth of the error with range. Typical errors are 10, 50, and 75% for realistic travel time errors (case 2) and three receivers at the three ranges 300, 600, and 1000 km. That this is primarily due to increasing range rather than data error can be seen in Figure 7. In this figure, $C_{\varepsilon_{RD}}$ is plotted versus data error for the three ranges and 1, 3, and 5 receivers. For a given data error, $C_{\varepsilon_{RD}}$ clearly increases with increasing range. Because the correlation length scale $L_{c}$ is held constant at 100 km, there are an increasing number of eddies that the inverse procedure must account for as the range is increased.

The error in the range-averaged sound speed perturbation $C_{\varepsilon_{RA}}$ is plotted versus data error in Figure 8. Just two additional receivers reduce the error typically by a factor of 4. For the same reasons as were stated above, adding yet another two receivers does not have such a

<table>
<thead>
<tr>
<th>TABLE 2. Sound Speed and Current Velocity RMS Errors in Meters per Second</th>
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<tbody>
<tr>
<td>Number of Receivers</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>0 (a priori)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>3</td>
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<tr>
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<tr>
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<td>3</td>
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<tr>
<td>5</td>
</tr>
</tbody>
</table>

$C_{\varepsilon_{RA}}$, $u_{\varepsilon_{RD}}$, $u_{\varepsilon_{RA}}$
pronounced effect. Reducing the data error has a similar effect. Also, the larger the range, the less the error, since more eddies are being averaged together. For all cases the error is less than 0.07%, and in most cases it is less than 0.01%. This is equivalent to approximately 0.01 m s\(^{-1}\) or 0.002°C rms uncertainty averaged over the entire slice or 10 times these values at 1000-m depth where the first baroclinic sound speed mode \(Z_1\) has its maximum value (this error would be less if axial rays were used).

Current Velocity

Increasing the number of receivers has little effect on the range-dependent velocity error \(e_{\hat{h}D}\). There is a minimal effect of at most 5% as the data error was reduced. Typical values for \(e_{\hat{h}D}\) are 75, 85, and 95% for the three ranges 300, 600, and 1000 km. No real improvement should be expected until the absolute error for the sound speed field drops below the a priori level for current velocity, i.e., \(e_{\hat{h}D} \approx 0.4\%\), or unless multiple sources (transceivers) are used.

In a like fashion, adding receivers has no effect on reducing the range average error \(e_{\hat{h}A}\). Given the relatively low expected signal-to-noise ratio (~a priori variance/data error), the additional data have no new independent information. Reducing the data error, though, does reduce \(e_{\hat{h}A}\) significantly (Figure 9). A typical value is 0.1% in variance or 2 mm s\(^{-1}\) rms.

5. DISCUSSION

The error in these simulated tomography experiments is a function of three parameters: (1) the number of receivers used and the associated ray paths, (2) the travel time data error, and (3) the range \(R\). Although the horizontal correlation length scale \(L_x\) was held fixed, its effect must also be considered. The primary purpose of this work was to evaluate the effect of the range-dependent error of adding receivers. The ocean slice becomes more completely sampled by increasing the number of receivers and rays. With enough receivers no part of the ocean slice larger than the smallest significant length scale is left unsampled by the acoustic rays. The additional data are independent only if the data error is small enough. For the 300-km case in Figure 10, \(e_{\hat{h}D}\) does not decrease with decreasing data error for the one receiver case because some parts of the ocean have been left unsampled (see Figure 1b). Even for very small data error, \(e_{\hat{h}D}\) will not decrease significantly because of incomplete sampling. Once multiple receivers are used, the error drops significantly. For the larger ranges, additional receivers help only if the data error is small.

Another factor controlling the error (both range-averaged and range-dependent) is the correlation length scale \(L_x\) relative to the range. The ratio of \(L_x\) to the range controls the partitioning of the a priori and error variances into range-averaged and range-dependent components. For a larger \(L_x\) than was used here (100 km), the variance will be redistributed to the mean where tomography does a better job, and the percentage range-dependent error will be less.

Near each end of the slice, more structure is visible in the ray paths, especially in the first half ray loop (Figure 3a). This structure has the effect of lowering the error.
near the receivers. In Figure 10, \( \epsilon(x, z) \) is plotted for three receivers, data case 2, and \( R = 600 \) km. It is seen that the estimated error at each mooring is quite small, similar to what one might expect from point measurements. If in fact, only point measurements of \( \delta C \) are made at each end of the slice, the error field obtained is shown in Figure 11. (The particular error levels chosen for the point measurements, \(-0.05 \) m s\(^{-1}\) or \(0.01 ^\circ \text{C} \) rms, were obtained by assuming a Garrett-Munk internal wave spectrum averaged over 1 day at a latitude of 30°.) The point measurement errors (Figure 11) and the tomographic errors (Figure 10) are approximately the same at each mooring, indicating that adding the point measurements to the tomographic measurement would not significantly reduce the errors.

The primary difference between the two error fields is in the interior of the region beginning about one correlation length from each mooring. The error for the point measurement case increases to the a priori error in the interior of the slice, while the tomographic error is reduced overall from the a priori value. This reduction of error in the interior is due to the efficient tomographic measurement of the mean. These figures show that for reasonable data errors and ranges greater than, say, 500 km, a single-slice tomographic system can be regarded as a measuring device producing both a good estimate of the mean field and an estimate approaching that of point measurements at each end of the slice.

Further, these results suggest that a mooring with acoustic receivers in the middle of the slice (at 300 km in this case) would reduce the error even further.

**Conclusions**

The model used in these vertical slice tomographic simulations is necessarily a simple one. In reality, the model must be made more complicated with more degrees of freedom to include effects of fronts, surface mixed layers, bottom boundary layers, anomalous water masses, gyre scale flow with longer time scales and different vertical structure, etc. As a result, errors in a real situation may be somewhat larger, but the trends shown here should hold and they can be summarized as follows: (1) Multiple receivers in a single-slice tomographic experiment improve the range-dependent perturbation sound speed estimates, with the degree of improvement dependent on the range between instruments. At 300 km, \( \epsilon_{\delta D} = -10\% \) for a reasonable data error but grows rapidly to \(-75\% \) at 1000 km. Reducing the latter will depend both on how the nonlinearities in the travel times are treated and on the incorporation of additional independent measurements. Adding an even larger number of receivers (possibly a line receiver), and thus more rays and data, would reduce the error in a \(1/N\) fashion. (2) The range average error \( \epsilon_{\delta D} \) is significantly reduced by typically a factor of 4 with multiple receivers, but in this case the longer range cases have the smaller errors. A typical value is \(0.02\% \) or \(0.01 \text{ m s}^{-1}\) rms. This reduction in the range-averaged error alone may justify using multiple receivers to obtain better estimates of mean ocean temperatures and trends with time. (3) The current velocity errors \( \epsilon_{\delta A} \) and \( \epsilon_{\delta D} \) are insensitive to the number of receivers. Only \( \epsilon_{\delta D} \) is decreased by reducing the data error; a realistic value is \(0.1\% \) or \(2 \text{ mm s}^{-1}\) rms. The range-dependent error \( \epsilon_{\delta D} \) will remain high until \( \epsilon_{\delta D} \) is reduced to a value less than the a priori current velocity variance, or unless additional sources are used.

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